Working Paper

EQUILIBRIUM SELECTION IN COMMON-VALUE SECOND-PRICE AUCTIONS

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This note considers equilibrium selection in common-value second-price auctions with two bidders. We show that for each ex post equilibrium in continuous and undominated strategies, a sequence of "almost common-value" auctions can be constructed such that each of them possesses a unique undominated and continuous equilibrium and the corresponding sequence of equilibria converges to that ex post equilibrium. As an implication, no equilibrium selection of this model based on perturbations seems to be more convincing than others.

1. INTRODUCTION

The well-known linkage principle (Milgrom and Weber (1982)) in auction theory states that the expected revenue in the symmetric equilibrium of a second-price auction is no less than the expected revenue from a first-price auction. However, Milgrom (1981) uses a simple example to illustrate that there could be a continuum of *asymmetric* equilibria in common-value second-price auctions, which are not revenue-equivalent. In fact, it is easy to see that the seller's revenue in an asymmetric equilibrium can be very low. Thus, multiplicity of equilibria generates difficulties for revenue comparisons in common-value auctions. While in his original work Milgrom calls these asymmetric equilibria "strange," a subsequent study by Klemperer (1989) suggests that asymmetric equilibria may be the only reasonable ones in the sense that by giving a slight advantage to one bidder, almost all equilibria are "extreme" as the advantaged bidder wins the auction with probability one in any undominated and continuous equilibrium. Therefore, there seems to be no obvious reason to favor the symmetric equilibrium over asymmetric ones.

In response to this multiplicity problem, various studies have been devoted to selecting a particular equilibrium in the second-price auction by perturbing the model in different ways.[1] Parreiras (2006) perturbs the second-price auction format to a hybrid auction involving the winner paying the highest bid with a small probability and the second highest bid with the complementary probability. He shows that the hybrid auction generates at least as much revenue as the first-price auction when signals are affiliated, thereby providing a justification for the linkage principle. Abraham et al. (2012) define a notion of tremble-robust equilibrium based on the idea

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[1] There is also an alternative approach to equilibrium selection in auctions based on iterated elimination of dominated strategies. See Harstad and Levin (1985) for an example.

that there is a small probability that an additional bidder may be present in the auction and draws her bid according to a predetermined smooth distribution.[2] In a model with asymmetrically-informed bidders, they select the equilibrium that generates the lowest revenue for the seller. Cheng and Tan (2008) provide a justification of the symmetric equilibrium by adding a small private-value component to the common-value model. Larson (2009) also considers private-value perturbations. He shows that by adding a private-value component that is independent of the common-value signals, asymmetric perturbations lead to selections of asymmetric equilibria under certain restrictions on the common-value component and signal distributions.

In contrast to the previous literature, this note provides a general analysis of equilibrium selection in pure common-value second-price auctions. For such auctions, we provide a negative conclusion to the approach of equilibrium selection based on payoff perturbations. In particular, we show that every increasing and continuous equilibrium can be selected by perturbing bidders' valuations in a certain manner. An implication of this result is that symmetric equilibria can only survive symmetric perturbations of payoffs. Similar results hold in second price auctions with more than two bidders and in English auctions more generally.

While the main results apply to equilibria in monotone and continuous strategies, we also identify a class of equilibria in discontinuous and undominated strategies that may not be monotone in common-value second-price auctions.[3] However, we show that all those discontinuous equilibria are fragile to the introduction of a noisy bid. In contrast, all equilibria in continuous and undominated strategies are robust to this perturbation, thereby justifying our focus on the continuous equilibria.

2. A COMMON-VALUE SECOND-PRICE AUCTION WITH TWO BIDDERS

Consider a pure common-value auction with two bidders. There is a single object for sale and two risk-neutral bidders compete for the object via a sealed-bid second price auction. The value V of the object is the same to both bidders. Prior to submitting bids, each bidder receives a private signal that partially reveals the value of the object. For each $i \in \{1,2\}$, let $\tilde{s}_i$ denote bidder i's private signal. Assume that $(\tilde{s}_1, \tilde{s}_2)$ is drawn according to the cumulative distribution function F with support $[0,1] \times [0,1]$. For each $i, j \in \{1,2\}$ and $i \neq j$, let $F_i(\cdot|s_j)$ denote the distribution of s_i conditional on bidder j's signal realization s_j. Assume that $F_i(\cdot|s_j)$ admits a density function $f_i(\cdot|s_j)$ that is strictly positive on $[0,1]$. The expected value of the object conditional on the signal pair (s_1, s_2) is given by $v(s_1, s_2) \equiv \mathbb{E}[V|s_1, s_2]$. Finally, assume that v is continuously differentiable and strictly increasing in each s_i.

[2] Equilibrium selection via the introduction of a noisy bidder is first considered by Hashimoto (2010) in a complete information generalized second-price auction.

[3] Birulin (2003) points out that there exist undominated *ex post* equilibria in discontinuous strategies when the auction admits an efficient *ex post* equilibrium.

A pure strategy for bidder i is a map $\beta_i : [0,1] \to \mathbb{R}$, which determines her bid for any signal. We will consider equilibria in undominated pure strategies. Since there are two bidders, this model is equivalent to an English (open ascending-price) auction. It is well-known that this pure common-value auction has multiple equilibria.[4] The following class of undominated *ex post* equilibria is identified by Milgrom (1981).

LEMMA 2.1 *For every strictly increasing and onto function $h : [0,1] \to [0,1]$, the strategy profile $\beta_1(s_1) = v(s_1, h^{-1}(s_1))$ and $\beta_2(s_2) = v(h(s_2), s_2)$ is an undominated* ex post *equilibrium. Furthermore, all undominated* ex post *equilibria in continuous and increasing strategies are of this form.*

PROOF: See Milgrom (1981) and Bikhchandani and Riley (1991). □

Note that the seller's revenue in an asymmetric equilibrium can be very low. For example, consider the function $h(s) = s^\alpha$ where α is a constant. For large α, bidder 1's bids are close to $v(s_1, 0)$ with high probability. Since the losing bid determines revenue in a second-price auction, the seller's expected revenue is close to $\mathbb{E}[v(s_1, 0)]$ in this asymmetric equilibrium.

Unlike prior studies that select a particular equilibrium in second-price auctions (especially the symmetric equilibrium), in the next section we obtain a negative answer to equilibrium selection based on perturbations. Our results suggest that all these asymmetric equilibria are equally convincing.

3. EQUILIBRIUM SELECTION BY PRIVATE-VALUE PERTURBATIONS

Consider the following class of "almost common-value" second-price auctions. Let $\mathcal{H}$ denote the collection of all strictly increasing and continuous functions that map $[0,1]$ onto $[0,1]$. For each $h \in \mathcal{H}$ and each $\varepsilon \in (0,1)$, define the corresponding second-price auction $\Gamma^{\varepsilon,h}$ by perturbing the ex post payoff functions of both bidders to

$$\tilde{v}_1^\varepsilon(s_1, s_2) = \varepsilon s_1 + (1-\varepsilon) v(s_1, s_2)$$
$$\tilde{v}_2^\varepsilon(s_1, s_2) = \varepsilon h(s_2) + (1-\varepsilon) v(s_1, s_2).$$

Suppose that bidder 2 follows a monotone bidding function β_2, then bidder 1 with signal s_1 will bid b in order to maximize

$$\int_0^{\beta_2^{-1}(b)} \left[\varepsilon s_1 + (1-\varepsilon) v(s_1, s_2) - \beta_2(s_2)\right] f_2(s_2|s_1) ds_2.$$

[4] Milgrom (1981) first points out the multiplicity of ex post equilibria in common-value second-price auctions. Bikhchandani and Riley (1991) argue that there is a much larger class of perfect Bayesian equilibria in English auctions with more than two bidders.

The corresponding first-order condition is

$$\left[\varepsilon s_1 + (1-\varepsilon)v(s_1, \beta_2^{-1}(b)) - b\right] f_2(\beta_2^{-1}(b)|s_1)\beta_2^{-1\prime}(b) = 0.$$

Substituting b with the bid $\beta_1(s_1)$, we get

$$\varepsilon s_1 + (1-\varepsilon)v(s_1, \beta_2^{-1}(\beta_1(s_1))) - \beta_1(s_1) = 0. \tag{1}$$

Similarly, given bidder 1's bidding function β_1, the corresponding first-order condition from bidder 2's maximization problem yields

$$\varepsilon h(s_2) + (1-\varepsilon)v(\beta_1^{-1}(\beta_2(s_2)), s_2) - \beta_2(s_2) = 0. \tag{2}$$

Consider any bid $b = \beta_1(s_1)$ for some $s_1 \in [0,1]$. If there is some $s_2 \in [0,1]$ such that $\beta_2(s_2) = b$, then $s_1 = \beta_1^{-1}(\beta_2(s_2))$ and $s_2 = \beta_2^{-1}(\beta_1(s_1))$. From (1) and (2), it follows that

$$\varepsilon s_1 = b - (1-\varepsilon)v(s_1, s_2) = \varepsilon h(s_2).$$

Thus, a tie happens whenever the signal pair (s_1, s_2) is such that $s_1 = h(s_2)$. Moreover, β_1 and β_2 satisfy

$$\beta_2^{-1}(\beta_1(s_1)) = h^{-1}(s_1), \quad \forall s_1 \in [0, \bar{s}_1], \tag{3}$$

$$\beta_1^{-1}(\beta_2(s_2)) = h(s_2), \quad \forall s_2 \in [0, \bar{s}_2]. \tag{4}$$

Therefore, by (1)–(4), an equilibrium $\beta^\varepsilon = (\beta_1^\varepsilon, \beta_2^\varepsilon)$ of the perturbed second-price auction $\Gamma^{\varepsilon,h}$ must satisfy

$$\beta_1^\varepsilon(s_1) = \varepsilon s_1 + (1-\varepsilon)v(s_1, h^{-1}(s_1)), \tag{5}$$

$$\beta_2^\varepsilon(s_2) = \varepsilon h(s_2) + (1-\varepsilon)v(h(s_2), s_2). \tag{6}$$

Note that the private-value components enter both bidders' bidding functions. This follows from the fact that the price paid by the winning bidder does not depend on her own bid in second-price auctions. Existence and uniqueness of a continuous equilibrium in the perturbed auction follow directly from the analysis above. In fact, the next result also establishes that the equilibrium is *ex post*.

PROPOSITION 3.1 *In the perturbed auction $\Gamma^{\varepsilon,h}$, there exists a unique undominated* ex post *equilibrium in continuous strategies, which is characterized by (5) and (6).*[5]

[5]This equilibrium outcome is efficient in the perturbed auction. As we pointed out before, there are also discontinuous equilibria in the perturbed auction, but all those equilibria are inefficient.

PROOF: The first-order necessary conditions lead to a unique candidate profile (5) and (6) for a Bayesian Nash equilibrium. We now argue that this strategy profile is indeed an *ex post* equilibrium.

Suppose that the realization of signals (s_1, s_2) is such that $\beta_1^\varepsilon(s_1) > \beta_2^\varepsilon(s_2)$, then bidder 1 wins the auction and pays $\beta_2^\varepsilon(s_2)$. Since the bidding strategies β_1^ε and β_2^ε are increasing functions, and ties occur at $\tilde{s}_1 = h(\tilde{s}_2)$, it follows that $s_1 > h(s_2)$. As v is strictly increasing, bidder 1's ex post payoff is

$$\begin{aligned}&\varepsilon s_1 + (1-\varepsilon)v(s_1, s_2) - \beta_2^\varepsilon(s_2)\\ &\quad = \varepsilon(s_1 - h(s_2)) + (1-\varepsilon)(v(s_1, s_2) - v(h(s_2), s_2)) > 0.\end{aligned}$$

Therefore, bidder 1 gets a positive surplus. Moreover, because she cannot affect the payment in a second-price auction, bidding $\beta_1^\varepsilon(s_1)$ is an ex post best response to $\beta_2^\varepsilon(s_2)$. On the other hand, bidder 2, who loses the auction and gets a payoff of zero, cannot obtain a positive payoff by bidding more than $\beta_1^\varepsilon(s_1)$, since

$$\begin{aligned}&\varepsilon h(s_2) + (1-\varepsilon)v(s_1, s_2) - \beta_1^\varepsilon(s_1)\\ &\quad = \varepsilon(h(s_2) - s_1) + (1-\varepsilon)(v(s_1, s_2) - v(s_1, h^{-1}(s_1))) < 0.\end{aligned}$$

Therefore, bidding $\beta_2^\varepsilon(s_2)$ is an ex post best reply to $\beta_1^\varepsilon(s_1)$ for bidder 2. The case in which the signal realization (s_1, s_2) satisfies $\beta_1^\varepsilon(s_1) \leq \beta_2^\varepsilon(s_2)$ follows from a similar argument. □

REMARK 3.2 Bikhchandani and Riley (1991) remark that in the pure common-value auction, the first-order conditions only pin down a single equation for two unknown bids. This in turn leads to the existence of a continuum of equilibria. In light of their discussion, we find that the presence of a private-value component gives another equation in the two unknown bids, which results in equilibrium uniqueness in the perturbed auction.

As ε vanishes, the equilibrium $\beta^\varepsilon = (\beta_1^\varepsilon, \beta_2^\varepsilon)$ in the perturbed auction $\Gamma^{\varepsilon,h}$ converges to one of the equilibria in the common-value auction Γ. That is, as ε goes to 0, the bidding strategies (3) and (4) converge to

$$\beta_1(s_1) = v(s_1, h^{-1}(s_1)), \quad \text{and} \tag{7}$$

$$\beta_2(s_2) = v(h(s_2), s_2), \tag{8}$$

respectively. Therefore, in the common-value second-price auction, every ex post equilibrium that has the form (7) and (8) where $h \in \mathcal{H}$ is the limit equilibrium of a sequence of perturbed "almost common-value" auctions. Moreover, each perturbed "almost common-value" auction has a unique continuous and undominated equilibrium. The result is summarized in the next proposition.

Proposition 3.3 *For any undominated ex post equilibrium β of the common-value second-price auction Γ satisfying (7) and (8) for some $h \in \mathcal{H}$, there is a sequence of perturbed "almost common-value" second-price auctions $(\Gamma^{\varepsilon_k,h})_{k\in\mathbb{N}}$, where for each k, $\varepsilon_k \in (0,1)$, and $\lim_{k\to\infty}\varepsilon_k = 0$, such that each auction $\Gamma^{\varepsilon_k,h}$ has a unique undominated and continuous Bayesian Nash equilibrium β^k, and the corresponding sequence of equilibria $(\beta^k)_{k\in\mathbb{N}}$ converges to β as k goes to infinity.*

Remark 3.4 The above results hold under more general forms of private-value perturbations. For example, instead of considering s_1 and $h(s_2)$ as bidders' private values, we can assume that bidder i's payoff is $\varepsilon_k w_{i,k}(s_i) + (1-\varepsilon_k)v(s_1, s_2)$, where $\varepsilon_k \to 0$ as $k \to \infty$, and bidder i's private value component $w_{i,k}(s_i)$ may vary with k. Suppose that for each $i = 1, 2$ and each $k \in \mathbb{N}$, $w_{i,k}$ is a strictly increasing and continuous function that maps $[0,1]$ onto $[0,1]$. For each $k \in \mathbb{N}$ and each $s_2 \in [0,1]$, define $h_k(s_2) \equiv {w_{1,k}}^{-1}(w_{2,k}(s_2))$. Following the steps that lead to equations (5) and (6), for each k, the equilibrium of the perturbed auction is characterized by

$$\beta_1^k(s_1) = \varepsilon_k w_{1,k}(s_1) + (1-\varepsilon_k)v(s_1, h_k^{-1}(s_1)),$$

$$\beta_2^k(s_2) = \varepsilon_k w_{2,k}(s_2) + (1-\varepsilon_k)v(h_k(s_2), s_2)).$$

If for each pair (s_1, s_2), $(w_{1,k}(s_1), w_{2,k}(s_2))$ converges to $(s_1, h(s_2))$, then the sequence of functions $(h_k)_{k\in\mathbb{N}}$ converges pointwise to h. It follows that as k goes to infinity, the sequence of equilibria $(\beta^k)_{k\in\mathbb{N}}$ converges to β, which is given by (7) and (8).[6]

Proposition 3.3 gives a negative answer to the question of equilibrium selection in pure common-value second-price auctions. In particular, each asymmetric equilibrium in the common-value auction can be justified by a particular perturbation of the model. If all possible perturbations are equally likely ex ante, then it is not quite clear why one equilibrium is more appealing than the others. Therefore, without embedding the model into a larger context, the analysis of common-value second-price auctions based on selecting a particular equilibrium is rather incomplete.

On the other hand, the class of perturbed auctions considered in this note provides an explanation of the potential sources of the multiplicity. If one takes the viewpoint that pure common-value auctions are rare in real world situations, then the private-value component considered here can be interpreted as a reduced-form modeling of other factors that are relevant to bidders. The results in this note suggest that, as bidders put more weight on their common-value components ($\varepsilon \to 0$), the unique efficient equilibrium in the "almost common-value" auction leads to a unique prediction of the bidding behavior in the limiting common-value model.

[6] I thank an anonymous referee for suggesting this kind of perturbations.

REMARK 3.5 The same equilibrium selection results carry over to common-value auctions with more than two bidders or asymmetrically informed bidders (cf. Liu (2013)).

4. DISCONTINUOUS EQUILIBRIA IN SECOND PRICE AUCTIONS

The preceding analysis has focused on equilibria in continuous strategies. But the common-value second-price auction also possesses equilibria in discontinuous strategies, which may not even be monotone. In this section, we introduce another perturbation of the common-value auction following Abraham et al. (2012) and Hashimoto (2010) and provide a justification of our interest in continuous equilibria based on a new robustness notion. In particular, we show that all those discontinuous equilibria are fragile to the presence of a noisy bid. In contrast, all continuous equilibria studied in Section 2 and 3 are robust to this perturbation. The analysis here pertains to the two-bidder auction, but the results can be generalized to the n-bidder case.

Equilibria in discontinuous strategies can be constructed from any continuous equilibrium. For expositional simplicity, we consider the symmetric equilibrium $\beta = (\beta_1, \beta_2)$ where $\beta_i(s_i) = v(s_i, s_i)$ for $i = 1, 2$. Pick any $s', s'' \in (0, 1)$ with $s' < s''$. The following discontinuous strategies for the two bidders

$$\tilde{\beta}_1(s_1) = \begin{cases} v(s_1, s'') & \text{if } s_1 \in [s', s''] \\ v(s_1, s_1) & \text{otherwise} \end{cases}$$

$$\tilde{\beta}_2(s_2) = \begin{cases} v(s', s_2) & \text{if } s_2 \in [s', s''] \\ v(s_2, s_2) & \text{otherwise} \end{cases}$$

form an *ex post* equilibrium. Since s' and s'' are chosen arbitrarily, there is a continuum of equilibria in discontinuous strategies. Moreover, non-monotone equilibria can be constructed from the above discontinuous equilibrium by "twisting" the bids for $s_1, s_2 \in [s', s'']$. For example, pick any $\hat{s} \in (s', s'')$, it is straightforward to see that the strategy profile

$$\hat{\beta}_1(s_1) = \begin{cases} v(s'', s'') & \text{if } s_1 \in [s', \hat{s}) \\ v(s_1, s'') & \text{if } s_1 \in [\hat{s}, s''] \\ v(s_1, s_1) & \text{otherwise} \end{cases}$$

$$\hat{\beta}_2(s_2) = \tilde{\beta}_2(s_2)$$

is also an *ex post* equilibrium. In fact, for bidder 1 with signal $s_1 \in [s', s'']$, any bid in the interval $(v(s', s''), v(s'', s''))$ can be supported in an equilibrium. Likewise, for bidder 2 with signal $s_2 \in [s', s'']$, any bid strictly between $v(s', s')$ and $v(s', s'')$ can be an equilibrium bid.

The intuition for the fragility of discontinuous equilibria to noisy bids is simple. While those discontinuous equilibria are undominated, they are in some sense close to being dominated. In any such discontinuous equilibrium, one bidder sometimes bids aggressively as she believes that her opponent would submit low bids. The presence of a noisy bid could cause her to pay more than the value of the object and hence make her more cautious. Therefore, no equilibrium in discontinuous strategies is the limit of equilibria in perturbed auctions with noisy bids.

Formally, suppose each bidder i believes that with probability ε an additional bidder enters the auction and bids randomly according to a strictly positive density function g over the set of all possible bids. Let $\mathrm{BR}^{\varepsilon,g}(\beta_{-i}) : [0,1] \to \mathbb{R}_+$ denote the best response correspondence for bidder i to her opponent's strategy β_{-i}.

Definition 4.1 An equilibrium $\beta = (\beta_1, \beta_2)$ of the common-value second-price auction is said to be *robust to noisy bids* if for each $i = 1, 2$, for each $s_i \in [0,1]$,

$$\lim_{\varepsilon \to 0} d\left(\beta_i(s_i), \mathrm{BR}^{\varepsilon,g}(\beta_{-i})(s_i)\right) = 0,$$

where $d(x, S) = \inf\{|x - y| : y \in S\}$ is the distance from a point x to a set S.

To see that equilibria in discontinuous strategies are not robust, first consider the strategy profile $(\tilde{\beta}_1, \tilde{\beta}_2)$ constructed above. For each $s_1 \in (s', s'']$, define

$$k(s_1) \equiv \mathbb{E}\left[v(\tilde{s}_1, \tilde{s}_2) | \tilde{s}_2 \leq s'', \tilde{s}_1 = s_1\right]$$

and

$$l(s_1) \equiv \max\left\{k(s_1), v(s', s'')\right\}.$$

Since v is strictly increasing in s_2, there exists $\delta > 0$ such that $\tilde{\beta}_1(s_1) - k(s_1) > \delta$ for each $s_1 \in (s', s'']$. The next result shows that $\tilde{\beta}_1(s_1)$ is bounded away from the set of best responses $\mathrm{BR}^{\varepsilon,g}(\tilde{\beta}_2)(s_1)$.

Proposition 4.2 *Given the perturbation g, for any $\varepsilon > 0$ and any signal $s_1 \in (s', s'']$, bidder 1 with signal s_1 strictly prefers to bid $l(s_1) < \tilde{\beta}_1(s_1)$ rather than $\tilde{\beta}_1(s_1)$, given bidder 2's strategy $\tilde{\beta}_2$.*

Proof: Conditional on the event that the noisy bid is less than bidder 2's bid, bidder 1 is indifferent between $v(s', s'')$ and $\tilde{\beta}_1(s_1)$, since conditional on winning with either bid, bidder 1's payment is determined by $\tilde{\beta}_2(s_2)$.

On the other hand, if the noisy bid is larger than bidder 2's bid, then conditional on winning, bidder 1 has to pay the noisy bid. Since bidder 2 never submits bids between $v(s', s'')$ and $v(s'', s'')$ under $\tilde{\beta}_2$, any payment $p \in (l(s_1), \tilde{\beta}_1(s_1))$ indicates that the expected payoff of bidder 1 is negative, i.e.,

$$\mathbb{E}\left[v(\tilde{s}_1, \tilde{s}_2) | \tilde{s}_2 \leq s'', \tilde{s}_1 = s_1\right] - p = k(s_1) - p < k(s_1) - l(s_1) \leq 0.$$

Since the density of the noisy bid g has full support by assumption, the noisy bid lies in $(l(s_1), \tilde{\beta}_1(s_1))$ with positive probability. Therefore, it is never optimal for bidder 1 with signal $s_1 \in (s', s'']$ to bid above $l(s_1)$ in the presence of the noisy bid. $\square$

The above proposition implies that the strategy profile $(\tilde{\beta}_1, \tilde{\beta}_2)$ does not meet our robustness criterion. The source of fragility comes from the discontinuities in both bidders' strategies, as the proof indicates. Therefore, the proof can easily be generalized to show that all discontinuous equilibria constructed along the line of $\tilde{\beta}$ or $\hat{\beta}$ are not robust to noisy bids. In contrast, the next result demonstrates the robustness of the class of continuous equilibria identified by Milgrom (1981). See Liu (2013) for a proof.

PROPOSITION 4.3 *For each strictly increasing and onto function $h \in \mathcal{H}$, the equilibrium of the second-price auction, (β_1, β_2), where $\beta_1(s_1) = v(s_1, h^{-1}(s_1))$ and $\beta_2(s_2) = v(h(s_2), s_2)$, is robust to noisy bids.*

We conclude with a discussion of the connection between our robustness notion and Selten's trembling-hand perfection (Selten (1975)).[7] Both notions share the feature of using full-support strategies to select certain equilibria. In normal form games, perfection rules out equilibria in weakly dominated strategies. In a second-price auction (a Bayesian game), our robustness notion excludes equilibria in discontinuous strategies. However, there is an important distinction between the two notions. While trembling-hand perfection considers totally mixed strategies through the perturbation of strategy sets, our robustness notion requires each bidder to believe that "any rival bid is possible" even if her opponent's strategy does not have full support.

The reason why we do not pursue strategy perturbations is that in our common-value environment, each bidder's strategy provides additional information about the value of the object to her opponents, and strategy perturbations would inevitably confound each bidder's inference problem. Noisy bids, on the other hand, circumvent these issues and are powerful enough to exclude all the "unintuitive" discontinuous equilibria.

REFERENCES

ABRAHAM, I., S. ATHEY, M. BABAIOFF and M. GRUBB (2012). Peaches, lemons, and cookies: Designing auction markets with dispersed information. *Working paper.*

BIKHCHANDANI, S. and J. RILEY (1991). Equilibria in open common value auctions. *Journal of Economic Theory* **53**(February), 101–130.

BIRULIN, O. (2003). Inefficient ex-post equilibria in efficient auctions. *Economic Theory* **22**(3), 675–683.

CHENG, H. and G. TAN (2008). Asymmetric common value auctions with applications to auctions with resale. *Working paper.*

[7] See also Simon and Stinchcombe (1995) for notions of perfection in infinite normal-form games.

Cheng, H. and G. Tan (2010). Asymmetric common value auctions with applications to auctions with resale. *Economic Theory* **45**(October), 253–290.

Engelbrecht-Wiggans, R., P. Milgrom and R. Weber (1983). Competitive bidding and proprietary information. *Journal of Mathematical Economics* **11**(2), 161–169.

Harstad, R. and D. Levin (1985). A class of dominance solvable common-value auctions. *The Review of Economic Studies* **52**(3), 525–528.

Hashimoto, T. (2010). Equilibrium selection, inefficiency, and instability in internet advertising auctions. *Working paper*.

Klemperer, P. (1989). Auctions with almost common values: The "wallet game" and its applications. *European Economic Review* **42**(3-5), 757–769.

Larson, N. (2009). Private value perturbations and informational advantage in common value auctions. *Games and Economic Behavior* **65**(2), 430–460.

Liu, H. (2013). Equilibrium selection in common-value second-price auctions. *Working paper* at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2345530.

Milgrom, P. (1981). Rational expectations, information acquisition, and competitive bidding. *Econometrica* **49**(4), 921–943.

Milgrom, P. and R. Weber (1982). A theory of auctions and competitive bidding. *Econometrica* **50**(5), 1089–1122.

Parreiras, S. (2006). Affiliated common value auctions with differential information: The two bidder case. *The B.E. Journals in Theoretical Economics* **6**(1), Article 12.

Selten, R. (1975). Reexamination of the perfectness concept for equilibrium points in extensive games. *International Journal of Game Theory* **4**(1), 25–55.

Simon, L. and M. Stinchcombe (1995). Equilibrium refinement for infinite normal-form games. *Econometrica* **63**(6), 1421–1443.