FALSE-NAME BIDDING IN REVENUE MAXIMIZATION PROBLEMS ON A NETWORK

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Abstract. This paper studies the allocation of several heterogeneous objects to buyers with multidimensional private information. Motivated primarily by airline-pricing problems, we impose certain substitution and complementarity assumptions on the buyers' preferences over bundles of objects. A novel incentive concern arises in this setting: buyers can false-name bid, i.e., one buyer submitting multiple bids as several different buyers. We study both static and dynamic revenue maximizing mechanisms that are false-name proof. Our main finding is that randomized mechanisms can dominate deterministic mechanisms in both static and dynamic settings, as the former can relax buyers' false-name proof constraints via randomized allocation rules, which in turn lead to higher revenues for the seller. Furthermore, in the dynamic setting, within the class of deterministic mechanisms, we show that false-name proofness justifies the consideration of "bid pricing" mechanisms.

Keywords: Multidimensional Mechanism Design, Revenue Maximization, False-name Proofness.

1. Introduction

In many monopoly-pricing problems, sellers often have heterogeneous products for sale. Moreover, buyers may treat products in a bundle as complements and different bundles as substitutes. For instance, in the airline industry, each airline company owns a flight network. It operates many different flights at the same time, most of which are interconnected. Moreover, buyers have different departure cities and destinations, and they can choose different routes to satisfy their demand. A buyer who wants travel from New York to Barcelona has many choices: she can buy either a direct flight or indirect connecting flights. Another such example is IKEA, where a buyer who wants to decorate a kitchen can either choose an already designed kitchen or buy pieces separately. In these problems, a buyer has multi-dimensional private information (the subsets of desired objects and the corresponding values), which creates novel and practical incentive concerns. One of them is *false-name bidding*. For example, United Airlines recently sued a cheap-airfare website named *Skiplagged*, as Skiplagged helps consumers find longer flights that include a stop in their

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destinations en route to other destinations.[1] This is exactly a form of false-name bidding studied in this paper.

In this paper, we provide a succinct framework to analyze revenue management problems in the presence of the heterogeneity described above. In particular, we model such heterogeneity using a directed graph where the edges of the graph represent different objects and the adjacency matrix of the graph describes the complementarities among the objects. The model captures the essential features of many real-life situations. Using the airline example mentioned before, each buyer's characteristics are represented by three parameters: her departure and destination cities and the value attached to her trip. We study both static and dynamic versions of this model, focusing on the implications of potential false-name bidding on the revenue maximizing mechanisms. We consider two types of incentive constraints. One is the standard incentive compatibility, which requires buyers to report their values truthfully; the other one is false-name proofness, which requires buyers to submit their reports using their true identities. Our main finding is that in the presence of false-name bidding, randomized mechanisms can dominate deterministic mechanisms in both static and dynamic settings, as the former can relax buyers' false-name proof constraints via randomized allocation rules, which in turn lead to higher revenues for the seller. Furthermore, in the dynamic setting, within the class of deterministic mechanisms, we show that false-name proofness leads to the consideration of "bid pricing" mechanisms, which have been extensively studied in operation research.[2]

In the static setting, we first give sufficient conditions on the distributions of buyers' values, under which buyers do not have incentive to false-name bid in the optimal (i.e., revenue-maximizing) deterministic mechanism. When the sufficient conditions do not hold, the false-name proof constraints may bind in the optimal mechanism. In this case, we consider specific examples and use the Lagrangian approach to show how false-name bidding affects the optimal allocation and transfers. In particular, we show that the optimal mechanism involves randomized allocation rules.

Next we consider a dynamic monopoly-pricing problem where the monopolist sells a limited number of heterogeneous objects before a certain deadline and short-lived consumers arrive over time. In the relaxed problem without the false-name bidding constraints, the optimal allocation rule is a cutoff rule: in each period and for each object, if a consumer arrives and demands that object, then she receives that object only if her value is above a certain cutoff, which is also the price

[1] http://techcrunch.com/2014/12/30/united-and-orbitz-sue-hidden-cities-flight-search-engine-skiplagged/

[2] Bid pricing refers to linear additive pricing of a bundle of goods. That is, the price of a bundle is equal to the sum of the prices of each good in this bundle. Bid pricing is first introduced in the operation research literature by Simpson (1989) and is further studied by Williamson (1992). They have advocated bid pricing for being computationally simple and efficient. More recently, Akan and Ata (2009) show that bid pricing is an ε-optimal policy in a continuous time revenue management problem in which demand is modeled as flow.

she needs to pay. The cutoffs for each object are deterministic and evolve over time, depending on not only the supply of this object, but also the supplies of complementary objects. We then show that under certain conditions, the optimal false-name proof mechanism can be implemented by posted price mechanisms in which prices are determined from the cutoffs. Specifically, the price of an object will increase whenever a substitute of that object is sold, and it will decrease whenever a complement of that object is sold. To put it differently, the heterogeneity among the objects allows for rich and intuitive price dynamics, which can be taken as a theoretical basis for further empirical investigations of the pricing patterns in various industries. Indeed, in the airline industry, there is strong empirical evidence (McAfee and te Velde, 2006, Lazarev, 2012) that ticket prices vary frequently in a non-monotone fashion. Our results give a partial explanation of this phenomenon: the ticket prices of each flight vary with the supplies of all connecting flights.

We then examine cases in which the optimal mechanisms cannot be implemented by posted prices due to binding false-name proof constraints. Using a two-period example, we show that dynamic allocations may mitigate some buyers' false-name bidding incentive, because of the option value of allocating the goods in the future. Perhaps more surprisingly, due to the complementarity among the objects, it is also possible that even if the false-name proof constraints do not bind in a static setting (in the second period), they become binding in dynamic settings (in the first period). Finally, our results also suggest that false-name bidding concerns offer a new justification for "bid pricing" mechanisms.

1.1. **Related Literature.** Revenue management problems have been studied extensively in both operation research and economics. Classic models in operation research usually consider myopic and non-strategic buyers, with focus on computational performances of dynamic allocations and pricing.[3] Within the operation research literature, our paper is closest to Talluri and van Ryzin (1998), which shows the sub-optimality of bid pricing in a network revenue management model. Our paper incorporate private information and false-name bidding behavior into the framework of Talluri and van Ryzin (1998).

The economics literature on revenue management stresses the implications of private information and strategic buyers and adopts mechanism design techniques to study revenue management.[4] Pai and Vohra (2013), Board and Skrzypacz (2013) and Mierendorff (2015) for instance, examine models with identical goods and strategic buyers. Gershkov and Moldovanu (2009) consider the

[3]See the monograph by Talluri and van Ryzin (2006) for an comprehensive survey of the operation research literature on revenue management.

[4]See Bergemann and Said (2011) and Vohra (2012) for excellent surveys on dynamic mechanism design. Gershkov and Moldovanu (2015) provide a comprehensive introduction of the mechanism design approach to study revenue management problems. Recent work on revenue management includes Pai and Vohra (2013a), Board and Skrzypacz (2013), Dizdar, Gershkov and Moldovanu (2011), Gershkov and Moldovanu (2009, 2012), and Dilme and Li (2012).

case of heterogeneous goods, but the goods are ordered according to a common quality parameter. Our model also features heterogeneous goods, but the ordering on objects is different from that in Gershkov and Moldovanu (2009).

Within static models, this paper is closely related to combinatorial auctions.[5] Similar to our paper, Abhishek and Hajek (2010) and Ledyard (2007) examine the optimal combinatorial auctions for the case of single-minded buyers. However, neither of the two papers considers false-name bidding. On the other hand, Yokoo et. al (2004) show that the Groves mechanisms are vulnerable to false-name bidding and give sufficient conditions under which the pivot mechanisms are false-name proof. Compared to Yokoo et al (2004), we focus on Bayesian mechanisms, and our sufficient conditions for false-name proofness are different. Arnosti et al (2015) also consider an auction setting for online display advertising and show that their auction format is false-name proof.[6] The main difference between this paper and Arnosti et al (2015) is that we consider the case of heterogeneous goods with only a partial order on the attractiveness of the goods and we show that this generates novel implications for the optimal mechanisms.

Finally, the literature on multidimensional screening (c.f. Rochet and Stole (2003) for a survey) has emphasized that optimal mechanism might feature randomization, when the buyer has multidimensional private information about her valuations.[7] In contrast, buyers in our model have one-dimensional valuations but they may report differently in another dimension—their types. And we identify a different reason for the dominance of randomized mechanisms.

2. The Static Problem

Suppose a monopolist sells three types of objects, AB, BC and AC, to buyers. The quantities of these objects are represented by a supply vector $(C_{AB}, C_{BC}, C_{AC}) \in \mathbb{N}^3$.

Objects. Objects and their corresponding quantities can also be described by a directed graph, $G(N, E)$, where N denotes the set of nodes and E denotes the set of edges, and a capacity function $C : E \to \mathbb{N}$.[8] That is, $N = \{A, B, C\}$, $E = \{AB, BC, AC\}$, and $C(\theta) = C_\theta$, for any $\theta \in E$.

Buyers. Each buyer's private information is a tuple (θ, v), where $\theta \in \Theta = \{AB, BC, AC\}$ is the buyer's type and v is her value. Specifically, a buyer's type θ represents her acceptance set: if she obtains any bundle of objects in her acceptance set then her payoff is v, otherwise her payoff is zero. For each θ, a type θ buyer's value v takes values from $V_\theta = [0, \bar{v}_\theta]$ and is distributed according

[5] For combinatorial auctions, Cramton (2006) is an excellent reference.

[6] In the literature false-name proof and shill-bidding proof are used interchangeably.

[7] For recent contributions on multidimensional screening, see also Thanassoulis (2004), Pycia (2006), Manelli and Vincent (2006, 2007), Briest, Chawla, Kleinberg, and Weinberg (2010), Pavlov (2011), and Hart and Renny (2013).

[8] The model can be easily generalized to the case with more types of objects using a graph representation. See Section 5.1 for further discussion. Talluri and van Ryzin (1998) use a similar representation.

to a cumulative distribution function G_θ with strictly positive density g_θ. Let $\Theta \times [0, \bar{v}]$ denote the set of private information, where $\bar{v} = \max_\theta\{\bar{v}_\theta\}$. We assume type AB buyers' acceptance set is $\{AB, \{AB, BC\}\}$, type BC buyers' acceptance set is $\{BC, \{AB, BC\}\}$, and type AC buyers' acceptance set is $\{AC, \{AB, BC\}\}$.

In the context of airline pricing, the nodes are three cities, the edges are the flights that the airline operates, and the capacities are the numbers of available tickets. A buyer's type is her departure city and destination, and her value is the payoff if she completes the trip. The above specification for the acceptance set implies that a buyer obtain the same payoff as long as she completes her trip.

In order to have non-trivial false-name bidding possibilities, we further assume that the number of each type buyers is random. Specifically, for each θ, the number of buyers, L_θ, takes values in $\mathbb{N}_+$ and is distributed according to a probability mass function l_θ, i.e., for any n, $l_\theta(n) \geq 0$ and $\sum_{n=1}^{\infty} l_\theta(n) = 1$.

Allocation and Payment. By the revelation principle, we focus on direct mechanisms.[9] A direct mechanism asks existing buyers to report their types and values, and then for all possible report profile the corresponding allocations and monetary transfers. For simplicity, we consider mechanisms that are symmetric across buyers of the same type. Formally, a mechanism $\langle p, t\rangle$ consists of an allocation rule a and a transfer rule t, such that for any realized numbers of buyers $(n_\theta)_\theta$, the allocation rule $p = (p_\theta)$ is defined as

$$p_\theta : \prod_{\theta \in \Theta} (V_\theta \times \{\theta\})^{n_\theta} \to [0, 1],$$

which is the probability that a type θ buyer's demand is satisfied, and the transfer rule $t = (t_\theta)$ is defined as

$$t_\theta : \prod_{\theta \in \Theta} (V_\theta \times \{\theta\})^{n_\theta} \to \mathbb{R},$$

which is payment made by a type θ buyer. Equivalently, for any given report profile $(\overrightarrow{v_{AB}}, \overrightarrow{v_{BC}}, \overrightarrow{v_{AC}})$, we can write the allocation as $p_\theta(\overrightarrow{v_{AB}}, \overrightarrow{v_{BC}}, \overrightarrow{v_{AC}})$ and the transfer as $t_\theta(\overrightarrow{v_{AB}}, \overrightarrow{v_{BC}}, \overrightarrow{v_{AC}})$.

For any mechanism $\langle p, t\rangle$, the corresponding interim allocation rule for a buyer with private information (v_θ, θ), $P_\theta(v_\theta, \theta)$, is given by $P_\theta(v_\theta, \theta) = \mathbf{E}\left[p_\theta(\overrightarrow{v_{AB}}, \overrightarrow{v_{BC}}, \overrightarrow{v_{AC}})\right]$. That is, $P_\theta(v_\theta, \theta)$ is the interim probability that a type θ buyer's demand is satisfied when she reports θ and v_θ. Similarly, the interim transfer rule for the buyer, $T_\theta(v_\theta, \theta)$, is given by $T_\theta(v_\theta, \theta) = \mathbf{E}\left[t_\theta(\overrightarrow{v_{AB}}, \overrightarrow{v_{BC}}, \overrightarrow{v_{AC}})\right]$.

[9]Yokoo et. al. (2004) first point out that the revelation principle still holds in the presence of false-name bidding. Since our setting is slightly different from that in Yokoo et. al. (2004), we provide a formal proof of the revelation principle in the Online Appendix.

Finally, to formally introduce false-name bidding, we denote $P_\theta(v_{\theta'}, \theta')$ the interim probability that a type θ buyer's demand is satisfied when she reports as a type θ' buyer and value $v_{\theta'}$.

Constraints. We study Bayesian incentive compatible (BIC) mechanisms. Given a mechanism $\langle p, t\rangle$, there are two kinds of incentive constraints of the buyers: the first kind is the standard incentive compatibility conditions, which require buyers to report their values truthfully, provided that they report their types truthfully. That is, for any buyer with private information (v_θ, θ), and for any $v'_\theta \in V_\theta$,

$$v_\theta P_\theta(v_\theta, \theta) - T_\theta(v_\theta, \theta) \geq v_\theta P_\theta(v'_\theta, \theta) - T_\theta(v'_\theta, \theta). \qquad (IC)$$

The second kind is the false-name proof conditions, which says that a buyer prefers to reveal her true type rather than reporting as different types of buyers. In our setting, the relevant false-name proof (FN) conditions are the following: (1) for any $v_{AB} \in V_{AB}$,

$$v_{AB}P_{AB}(v_{AB}, AB) - T_{AB}(v_{AB}, AB) \geq \max_{v_{AC}}[v_{AB}P_{AB}(v_{AC}, AC) - T_{AC}(v_{AC}, AC)];$$

(2) for any $v_{BC} \in V_{BC}$,

$$v_{BC}P_{BC}(v_{BC}, BC) - T_{BC}(v_{BC}, BC) \geq \max_{v_{AC}}[v_{BC}P_{BC}(v_{AC}, AC) - T_{AC}(v_{AC}, AC)];$$

(3) for any $v_{AC} \in V_{AC}$,

$$v_{AC}P_{AC}(v_{AC}, AC) - T_{AC}(v_{AC}, AC) \geq \max_{v_{AB}, v_{BC}} \{v_{AC}P_{AB}(v_{AB}, AB)P_{BC}(v_{BC}, BC) - T_{AB}(v_{AB}, AB) - T_{BC}(v_{BC}, BC)\}.$$

The first condition requires that any AB type buyer does not want to pretend to be an AC type buyer, the second condition requires that any BC type buyer has no incentive to pretend to be an AC type buyer, and the third condition requires that any AC type buyer does not want to pretend to be one AB and one BC type buyers. Note that since the realized number of buyers is random, the seller may not detect false-name bids from the number of reports.

We also normalize each buyer's reservation payoff to be zero, so that the individual rationality constraints are: for any buyer with private information (v_θ, θ), and for any $v'_\theta \in V_\theta$,

$$v_\theta P_\theta(v_\theta, \theta) - T_\theta(v_\theta, \theta) \geq 0. \qquad (IR)$$

Finally, the feasibility constraint (F) for the allocation rule p is: for any realized numbers of buyers $(n_\theta)_\theta$,

$$n_{AB}p_{AB} \leq C_{AB},\ n_{BC}p_{BC} \leq C_{BC},\ \text{and}\ n_{AC}p_{AC} \leq C_{AC} + \min\{C_{AB} - n_{AB}p_{AB}, C_{BC} - n_{BC}p_{BC}\}.$$

Revenue Maximization. The monopolist's problem is to look for a mechanism $\langle p, t \rangle$ that maximizes expected revenue subject to (IC), (FN), (IR) and (F). To solve the problem, we first consider the relaxed problem without the false-name proof constraints. Then we impose conditions on the primitives so that the solution to the relaxed problem is false-name proof. In the next section, we will consider the implications of binding false-name proof constraints.

We impose the following regularity condition (Assumption 1) so that the relaxed maximization problem can be solved using the classic Myerson approach (Myerson (1981)).

Assumption 1. For any $\theta \in \{AB, BC, AC\}$, the conditional virtual value function

$$\psi_\theta(v) \equiv v - \frac{1 - G_\theta(v)}{g_\theta(v)}$$

is non-decreasing in v.

Assumption 1 is the standard regularity condition for each type buyers' values, which gives the monotonicity of the allocation rule and hence the relaxed problem can be solved by point-wise maximization.

Then we provide two more conditions (Assumptions 2 and 3), which guarantee that the solution to the relaxed problem satisfies all the false-name bidding constraints (Proposition 1).

Assumption 2. The upper bounds of buyers' values satisfy: $\bar{v}_{AC} \geq \max\{\bar{v}_{AB}, \bar{v}_{BC}\}$. And for any $v \in V_{AB}$, $\psi_{AC}(v) \geq \psi_{AB}(v)$; for any $v \in V_{BC}$, $\psi_{AC}(v) \geq \psi_{BC}(v)$.[10]

Assumption 3. The conditional virtual value functions satisfy:

$$\psi_{AB}^{-1}(0) + \psi_{BC}^{-1}(0) \geq \bar{v}_{AC}.$$

Assumption 2 implies that AC type buyers' values dominate AB and BC type buyers' values in the sense of first order stochastic dominance. Assumption 3 on the other hand requires that the upper bound of AC type buyers' values is not too large. These assumptions are plausible in many environments, including the airline-pricing example.

Proposition 1. *Under Assumptions 1, 2, and 3, the solution to the relaxed problem satisfies all the false-name proof constraints, and hence it is also the solution to the monopolist's revenue maximization problem.*

Proof. Suppose the solution to the relaxed problem violates some buyer's false-name proof constraint. Since the solution to the relaxed problem has reserve price $\psi_\theta^{-1}(0)$ for each object θ, if an AC buyer pretends to be an AB and a BC buyer and satisfies her demand, she has to pay at least

[10]This assumption is equivalent to the hazard rate order if $V_{AB} = V_{BC} = V_{AB}$.

$\psi^{-1}_{AB}(0) + \psi^{-1}_{BC}(0)$, which is larger than her true value by Assumption 3. Therefore, either AB or BC buyers' false-name proof constraints must be violated.

However, if an AB buyer pretends to be an AC buyer, she may obtains an AC object, which does not satisfy her demand. Thus, the AB buyer must obtain the bundle with one unit AB and one unit BC when she pretends to be an AC buyer. By Assumption 2, $\psi_{AC}(v_{AB}) \geq \psi_{AB}(v_{AB})$, if the AB buyer's demand is satisfied when she reports her true type, then she pays weakly more by pretending to be an AC buyer. Thus, the only possibility to benefit from false-name bidding is that her demand is not satisfied when she reports her true type. However, in this case, the AB buyer has to pay more than her true value when her demand is satisfied by pretending to be an AC buyer, according to the solution to the relaxed problem. By the similar argument, no BC buyer can profitably false-name bid. This is a contradiction. □

Remark 1. Proposition 1 gives sufficient conditions under which false-name bidding is dominated by reporting the true type. Under Assumption 1, Assumptions 2 and 3 are also necessary if we impose a stronger notion of dominant false-name proofness, that is, regardless of other buyers' reports, a buyer has no incentive to false-name bid.

Remark 2. There are other conditions in the literature that guarantee the solution to the relaxed problem to be false-name proof (See Ausubel and Milgrom, 2002, Lehmann et al., 2006, and Sher, 2012). These conditions are substitute conditions imposed on buyers' preferences. However, in our model buyers' preferences do not satisfy these conditions, since there are both complements and substitutes.[11]

Remark 3. If Assumption 2 does not hold, then it is possible that an AB buyer (or a BC buyer) can benefit from false-name bidding, i.e., she may pretend to be an AC buyer in order to win both objects and discard the extra object BC (or AB). If Assumption 3 does not hold, then AC buyers' false-name proof constraints may be binding. In this case, we can use the Lagrangian relaxation approach, that is, we can define the generalized virtual values by adding the dual Lagrangian multipliers of binding constraints to the conditional virtual values. See Section 2.2 for the analysis of cases in which false-name proof constraints bind.[12]

2.1. **The Irregular Cases.** In this section, we study the optimal mechanism when some false-name proof constraints are binding. In order to highlight the distortion caused by the false-name

[11]Sher (2012) shows that in the case of mixture goods, finding optimal false-name bidding strategies for the buyer (cost minimization problem) is equivalent to finding the efficient allocation in the combinatorial auction (the winner determination problem).

[12]This approach was suggested in Pai and Vohra (2013a) without calculating the duals, and subsequently adopted by Pai and Vohra (2013b) and Mierendorff (2015).

bidding, we examine the following three cases. In the first two cases, we consider a setting where there is no direct competition among the same types of buyers: Case 1 focuses on AB type buyer's false-name bidding; Case 2 concerns AC type buyer's false-name bidding. In the third case, we turn to an auction setting and investigate AC type buyer's false-name bidding.

Case 1:

Assume that an AB type buyer can false-name bid, that is, he can report as an AC type buyer, but AC type buyers can not false-name bid. Suppose there is one AB good and one BC good. For simplicity, assume there are no BC buyers and there is only buyer who could either be of AB type or of AC type. We first provide a preliminary result (Proposition 2), which follows from the standard analysis a la Myerson (1981).

Proposition 2. *Incentive compatibility constraints (IC) in this case are characterized by: (1) for any $\theta \in \{AB, AC\}$, the payoff of a buyer with private information (v, θ) in the truth-telling equilibrium, $U(v, \theta)$, is given by*

$$U(v,\theta) = U(0,\theta) + \int_0^v P_\theta(v,\theta)dv,$$

and

$$v > v' \Rightarrow P_\theta(v,\theta) \geq P_\theta(v',\theta);$$

(2) for any v,

$$U(v, AB) \geq U(v, AC) \text{ and } U(0, AB) = U(0, AC).$$

Proof. See the Appendix. □

Suppose that with probability ρ the buyer's type is AC and with probability $1-\rho$ it is AB. Then the monopoly pricing problem becomes:

$$\int_0^{\bar{v}} \left[\rho p_{AC}(v)\psi_{AC}(v)g_{AC}(v) + (1-\rho)p_{AB}(v)\psi_{AB}(v)g_{AB}(v)\right] dv$$

subject to

$$\forall v, \int_0^v p_{AB}(s)ds \geq \int_0^v p_{AC}(s)ds.$$

We note that this constraint resembles second-order stochastic dominance. Let $\lambda(v)$ be the Lagrangian multiplier for the constraint. We then rewrite the problem as (see Appendix B for the derivation):

$$\max_{p_{AC}, p_{AB}} \rho \int_0^{\bar{v}} p_{AC}(v)\left[\psi_{AC}(v) - \phi_{AC}(v)\right] g_{AC}(v)dv + (1-\rho)\int_0^{\bar{v}} p_{AB}(v)\left[\psi_{AB}(v) + \phi_{AB}(v)\right] g_{AB}(v)dv$$

where $\phi_{AC}(v) = \frac{\Lambda(v)}{\rho g_{AC}(v)}$, $\phi_{AB}(v) = \frac{\Lambda(v)}{(1-\rho)g_{AB}(v)}$ and $\Lambda(v) \equiv \int_v^{\bar{v}} \lambda(s)ds$. Note that $\Lambda(v)$ is non-increasing in v. Define $\eta_{AC}(v) \equiv \psi_{AC}(v) - \phi_{AC}(v)$ and $\eta_{AB}(v) \equiv \psi_{AB}(v) + \phi_{AB}(v)$ as the generalized virtual values in the presence of false-name bidding. Since η_θ may not be an increasing function, ironing is needed.[13] Following Myerson's approach, for each $q \in [0,1]$ and $\theta \in \{AB, AC\}$,

$$h_\theta(q) = \eta_\theta\left(G_\theta^{-1}(q)\right) \quad \text{and} \quad H_\theta(q) = \int_0^q h_\theta(r)dr.$$

Let F_θ be the largest convex function on $[0,1]$ such that $F_\theta \leq H_\theta$. Convexity of F_θ implies that it is differentiable almost everywhere. Then define $f_\theta(q) = F'_\theta(q)$ for all differentiable points q; for non differentiable points extend the definition using right continuity. Then the ironed virtual value is $\eta^r_\theta = f_\theta \circ G_\theta$, which is increasing. We can then maximize the ironed virtual value point-wise:

$$\max_{p_{AB}, p_{AC}} \rho \int_0^{\bar{v}} p_{AC}(v)\eta^r_{AC}(v)g_{AC}(v)dv + (1-\rho)\int_0^{\bar{v}} p_{AC}(v)\eta^r_{AB}(v)\psi_{AB}(v)g_{AB}(v)dv.$$

Note that this problem is in linear in the allocation rule, therefore the optimal allocation rule is again deterministic and is characterized by two prices c_{AB} and c_{AC}. If false-name proof constraints bind, then in the optimal mechanism $c_{AB} = c_{AC}$; this implies that, comparing to the optimal mechanism without false-name bidding, more AB type buyers meet their demand, which may improve efficiency, but fewer AC type buyers meet their demand, which reduces efficiency; however, the overall efficiency comparison is ambiguous.

Case 2:

Next we consider the following case, in which an AC buyer can submit multiple bids as if she were an AB buyer and a BC buyer. We show that randomized mechanisms can improve upon any deterministic mechanisms.

Suppose the monopolist either meets with an AC buyer with probability 0.5, or meets with an AB buyer and a BC buyer with probability 0.5. Assume v_{AB} and v_{BC} are drawn independently from a uniform distribution over the interval $[0,1]$, and v_{AC} is drawn independently from a uniform distribution over the interval $[0,4]$.

In the optimal mechanism without false-name bidding, the monopolist posts prices: $\tilde{t}_{AB} = \tilde{t}_{BC} = 1/2$ and $\tilde{t}_{AC} = 2$. However this mechanism is not false-name proof, since any AC type buyer with value no less than one prefers to act as one AB and one BC type buyers and buy both AB and BC with a total expenditure of one.

[13] For the AC type we can give sufficient conditions which is g_{AC} is non-decreasing. For the AB type we cannot do so without knowing the $\Lambda(v)$.

In the optimal deterministic and false-name proof mechanism, the optimal posted prices t_{AB}, t_{BC} and t_{AC} are characterized by the binding false-name proof constraint:[14]

$$t_{AB} + t_{BC} = t_{AC}$$

and two first order conditions:

$$[1 - F_{AC}(t_{AB} + t_{BC}) - (t_{AB} + t_{BC})f_{AC}(t_{AB} + t_{BC})] + [1 - F_{AB}(t_{AB}) - t_{AB}f_{AB}(t_{AB})] = 0,$$

$$[1 - F_{AC}(t_{AB} + t_{BC}) - (t_{AB} + t_{BC})f_{AC}(t_{AB} + t_{BC})] + [1 - F_{BC}(t_{BC}) - t_{BC}f_{BC}(t_{BC})] = 0.$$

Given the distributions in this example, we have

$$t_{AB} = t_{BC} = \frac{2}{3}, \quad t_{AC} = \frac{4}{3}.$$

That is, the optimal false-name proof deterministic mechanism is a "bid pricing" mechanism. Comparing to the optimal mechanism without false-name proof constraints, now the prices for AB and BC are higher and the price for AC is lower, i.e., $t_{AB} > \tilde{t}_{AB}$, $t_{BC} > \tilde{t}_{BC}$ and $t_{AC} < \tilde{t}_{AC}$. To put it differently, fewer AB and BC type buyers will buy, and more AC type buyers will buy.

Next we consider a specific form of randomization and show that it can improve over the above optimal deterministic mechanism. Let $\pi_{AB} \in [0, 1]$ denote the probability that the AB good is never traded regardless of buyers' reported values. That is, the monopolist can commit to not selling AB with probability π_{AB}.[15] When $\pi_{AB} > 0$, the monopolist gives up surplus from AB buyers; but this relaxes the false-name proof constraint and hence generates more revenue from trading with AC buyers. That is, given any t_{AB}, t_{BC} and $\pi_{AB} \in (0, 1)$, the price t'_{AC} that the monopolist can charge for an AC buyer is

$$t'_{AC} = \frac{t_{AB} + t_{BC}}{1 - \pi_{AB}} \ (> t_{AC}).$$

In this case, the monopolist's expected revenue (as a function of π_{AB}) would be

$$W(\pi_{AB}) \equiv 0.5(1 - F_{AC}(t'_{AC}))t'_{AC} + 0.5[(1 - \pi_{AB})(1 - F_{AB}(t_{AB}))t_{AB} + (1 - F_{BC}(t_{BC}))t_{BC}].$$

[14]Note that in general if either $t_{AB} > \bar{v}_{AB}$ or $t_{BC} > \bar{v}_B C$, i.e., either AB or BC object is never allocated, then the false-name proof constraint will always hold.

[15]Alternatively, it is equivalent for the monopolist to charge a prohibitively high price for AB with probability π_{AB}. This interpretation is consistent with the empirical evidence in McAfee and te Velde (2006) that prices of airplane tickets may jump up very high occasionally.

Note that the revenue in the optimal deterministic mechanism is $W(0)$. Consider $W'(0)$, we have

$$\begin{aligned}
W'(0) &= (t_{AB}+t_{BC})(1-F_{AC}(t_{AB}+t_{BC})) - (t_{AB}+t_{BC})^2 f_{AC}(t_{AB}+t_{BC}) - (1-F_{AB}(t_{AB}))t_{AB} \\
&= -(t_{AB}+t_{BC})\left[1-F_{AB}(t_{AB}) - t_{AB}f_{AB}(t_{AB})\right] - (1-F_{AB}(t_{AB}))t_{AB} \\
&= \frac{10}{3}t_{AB} - 2 = \frac{2}{9} > 0,
\end{aligned}$$

where the second equality follows from the first order conditions. Therefore, the above randomized allocation (for some $\pi_{AB} > 0$) generates higher revenue than the optimal deterministic allocation for the monopolist.

The observation that randomized mechanisms improve upon deterministic mechanisms continues to hold in general settings. That is, the optimal randomized mechanism balances the trade-off between the benefit from charging a higher price to the buyer who may shill-bid and the cost of reducing the surplus to prevent shill-bidding. Moreover, such randomization leads to further inefficiency compare to the standard monopoly pricing problems without false-name bidding. To see this, consider the false-name proof constraint for an AC type buyer with value v

$$U_{AC}(v) \geq \max_{\tilde{v}_{AB},\tilde{v}_{BC}} p_{AB}(\tilde{v}_{AB},\tilde{v}_{BC})p_{BC}(\tilde{v}_{AB},\tilde{v}_{BC})v - t_{AB}(\tilde{v}_{AB},\tilde{v}_{BC}) - t_{BC}(\tilde{v}_{AB},\tilde{v}_{BC}),$$

from which we obtain a necessary condition

$$\int_0^v p_{AC}(s)ds \geq \int_0^v p_{AB}(\tilde{v}_{AB}(s),\tilde{v}_{BC}(s))p_{BC}(\tilde{v}_{AB}(s),\tilde{v}_{BC}(s))ds,$$

where $\tilde{v}_{AB}(\cdot)$ and $\tilde{v}_{BC}(\cdot)$ are the optimal false-name bidding strategy for the AC type buyer. Note that $\tilde{v}_{AB}(\cdot)$ and $\tilde{v}_{BC}(\cdot)$ depend on the mechanism (p,t). Since the AC type buyer can always report her true value v when false-name bidding, we have another weaker necessary condition that does not involve $\tilde{v}_{AB}(\cdot)$ and $\tilde{v}_{BC}(\cdot)$, i.e.,

$$\int_0^v p_{AC}(s)ds \geq \int_0^v p_{AB}(s,s)p_{BC}(s,s)ds.$$

Let $\lambda(v)$ be the associated Lagrange multiplier of the above constraint and temporarily ignore the monotonicity constraint. Using integration by parts, we rewrite the monopolist's problem as:

$$\begin{aligned}
&\max_{p_{AC},p_{AB},p_{BC}} \left\{\rho \int_0^{\bar{v}} p_{AC}(v)\psi_{AC}(v)f_{AC}(v)dv \right. \\
&+ (1-\rho)\left[\int_0^{\bar{v}} \bar{p}_{AB}(v)\psi_{AB}(v)f_{AB}(v)dv + \int_0^{\bar{v}} \bar{p}_{BC}(v)\psi_{BC}(v)f_{BC}(v)dv\right] \\
&\left. + \int_0^{\bar{v}} \Lambda(v)\left[p_{AC}(v) - p_{AB}(v,v)p_{BC}(v,v)\right]dv \right\},
\end{aligned}$$

where $\bar{p}_{AB}(v) = \int_0^{\bar{v}} p_{AB}(v,s) f_{BC}(s) ds$, $\bar{p}_{BC}(v) = \int_0^{\bar{v}} p_{BC}(s,v) f_{AB}(s) ds$, and $\Lambda(v) = \int_v^{\bar{v}} \lambda(s) ds$. Note that unless the allocation rule p_{AB} is independent of v_{BC} and p_{BC} is independent of v_{AB}, we can no longer maximize the above expression point-wise; nevertheless the above problem may have interior solutions due to the non-linear term $p_{AB}(v,v)p_{BC}(v,v)$. Therefore, the optimal mechanism may involve randomized allocations of either AB or BC when false-name bidding constraints bind, in order to reduce the AC type buyer's false-name bidding incentive. We also note that the allocation rule for AC remains a cutoff rule, where the cutoff is smaller than that in the relaxed problem in which there are no false-name bidding constraints, and the price for AC equals the cutoff. Unfortunately, we do not have a general characterization of the primitives that lead to randomization.

Case 3:

Suppose that the monopolist, who sells one unit of AB and one unit of BC, either meets with three buyers—one of each type—with probability 0.5, or meets with two AB buyers and two BC buyers with probability 0.5. Similar to Case 2, only the AC buyer can false-name bid. In particular, assume that whenever the AC buyer false-name bids, she reports $\tilde{v}_{AB} = \bar{v}_{AB}$ and $\tilde{v}_{BC} = \bar{v}_{BC}$. Consider the following generalization of the bid price mechanism in Case 2. A generalized bid price mechanism in this case consists of three reserve prices r_{AB}, r_{BC} and r_{AC} such that

- given a reported profile (v_{AB}, v_{BC}, v_{AC}),
  (1) if $\psi_{AC}(v_{AC}) > \max\{\psi_{AB}(v_{AB}) + \psi_{BC}(v_{BC}), r_{AC}\}$, then $p_{AC} = 1$ and
  $$t_{AC} = \inf\{\tilde{v}_{AC} \in V_{AC} : \psi_{AC}(\tilde{v}_{AC}) \geq \max\{\psi_{AB}(v_{AB}) + \psi_{BC}(v_{BC}), r_{AC}\}\}.$$
  (2) If $\psi_{AC}(v_{AC}) < \max\{\psi_{AB}(v_{AB}) + \psi_{BC}(v_{BC}), r_{AC}\}$, then $p_{AC} = 0$; in addition, if $\psi_{AB}(v_{AB}) > 0$, then $p_{AB} = 1$ and $t_{AB} = \psi_{AB}^{-1}(0)$, otherwise $p_{AB} = 0$; if $\psi_{BC}(v_{BC}) > 0$, then $p_{BC} = 1$ and $t_{BC} = \psi_{BC}^{-1}(0)$, otherwise $p_{BC} = 0$.
- given a reported profile $(v_{AB}^{(1)}, v_{AB}^{(2)}, v_{BC}^{(1)}, v_{BC}^{(2)})$, where $v_{AB}^{(1)} \geq v_{AB}^{(2)}$ and $v_{BC}^{(1)} \geq v_{BC}^{(2)}$, the two objects, AB and BC, are allocated separately in two second-price auctions with reserve prices r_{AB} and r_{BC}, respectively.

By the symmetry between AB and BC type buyers, we have $r_{AB} = r_{BC} \equiv r$. Note that the generalized bid price mechanism with reserve price $r = \psi_{AB}^{-1}(0)$ is the optimal mechanism for the relaxed problem without the false-name proof constraints.

Suppose for each buyer, v_{AB} and v_{BC} are drawn independently from a uniform distribution over the interval $[0,1]$, and v_{AC} is drawn independently from a uniform distribution over the interval $[0,4]$. If $r = \psi_{AB}^{-1}(0) = 0.5$ and $r_{AC} = \psi_{AC}^{-1}(0) = 2$, then the expected payoff of an AC buyer with value 2 from false-name bidding is:

$$2-\int_0^1 \max\{0.5, v_{AB}\}dv_{AB}-\int_0^1 \max\{0.5, v_{BC}\}dv_{BC}=2-1.25=0.75,$$

whereas her expected payoff from reporting her true type and value is zero. Therefore, the optimal mechanism for the relaxed problem is not false-name proof. This suggests that in the optimal false-name proof mechanism, we have $r \geq 0.5$ and $r_{AC} \leq 2$. However, unlike Case 2, because of the competition from existing AB and BC buyers reduces the AC buyer's incentive to false-name bid, the sum of the optimal reserve prices for AB and BC type buyers is strictly less than the optimal reserve price for the AC type buyer.

Finally, to clarify the logic that leads to randomized allocations, we conclude this section with the following example, which shows that even with degenerate values, randomization might still be necessary.

Example 1. Suppose there are one unit of AB and one unit of BC. The monopolist either meets with an AC buyer with probability ρ, or meets with an AB buyer and a BC buyer with probability $1-\rho$. Suppose all buyers have degenerate values: $v_{AB}=1$, $v_{BC}=3$, and $v_{AC}=5$. Then the optimal deterministic false-name proof mechanism requires AC to pay 4 and receive both objects for sure when $\rho \leq 0.5$. On the other hand, when $\rho > 0.5$, the optimal deterministic false-name proof mechanism never allocates the AB object; as a result the AC buyer's price is 5.

Now consider a randomized mechanism in which if a buyer reports AC then she gets both objects and pays 5; if a buyer reports BC she gets the object BC and pays 3; but if a buyer reports AB then with probability 0.75 she gets the object AB and pays 1, and with probability 0.25 she does not get the object AB and pays nothing. Under this mechanism, AC will never false-name bid, since doing so results a negative expected payoff.

Note that when $\rho \geq 0.5$, the randomized mechanism outperforms the optimal deterministic mechanism, i.e.,

$$(1-\rho)(0.75+3)+5\rho \geq \max\{4, 3+2\rho\} \quad \Leftrightarrow \quad 1 \geq \rho \geq 0.5.$$

In fact, one can verify that the above randomized mechanism is the unique optimal allocation mechanism whenever $\rho > 0.5$. Hence, in order to prevent profitable false-name bidding, the optimal mechanism sometimes requires randomization, which follows from the non-linearity of the false-name proof constraint. That is, the optimal randomized mechanism balances the trade-off between the benefit from charging a higher price to the buyer who may shill-bid and the cost of reducing the surplus to prevent shill-bidding. Moreover, such randomization leads to further inefficiency compare to the standard monopoly pricing problems without false-name bidding.

3. The Relaxed Dynamic Allocation Problem

In this section, we consider a dynamic version of the basic model. Suppose a monopolist sells three types of objects, AB, BC and AC, with supply vector $C_1 = (C_{AB}, C_{BC}, C_{AC}) \in \mathbb{N}^3$ over $S \geq 2$ periods. Time is discrete, labeled as $s = 1, 2, ..., S$, and there is no discounting. We assume that buyers arrive over time, each buyer has unit demand and is impatient, i.e., she needs to be served upon arrival. We first consider the case in which the monopolist observes which type of buyers arrive in every period, i.e., there are no false-name proof constraints. We will examine the case with false-name bidding in the next section.

Buyers. We assume that in each period there is only one buyer in the market. That is, at the beginning of each period s, a buyer arrives and observes the available capacities. The buyer privately learns her type $\theta \in \{AB, BC, AC\}$ and her value v: the probability of the type being θ is $\pi_\theta \geq 0$ and $\sum_\theta \pi_\theta = 1$, and given the buyer's type θ, her value v is drawn from the distribution G_θ. The buyer then reports her type and value; if she is not assigned any object in period s, she leaves the market and receives reservation payoff zero.

Histories. Let C_s denote the available supply in period s. Define h_s as all the reports up to period s and allocations up to period $s-1$, and $\bar{h}_s$ as all reports and allocation decisions up to and including period s. Let H_s be set of all possible histories in period s.

Revenue Maximization. A (direct) dynamic mechanism $\langle p, t \rangle$ consists of a sequence of allocation rules $p = (p_s)$ and a sequence of transfer rules $t = (t_s)$, where for each $s \in \{1, \ldots, S\}$, $p_s(h_s, \theta_s, v_s)$ is the probability that the buyer in period s satisfies her demand and $t_s(h_s, \theta_s, v_s)$ is the corresponding transfer.

In any period s, A type (θ_s, v_s) buyer's incentive compatibility constraint (IC) is: for any h_s and v'_s,

$$v_s p_s(h_s, \theta_s, v_s) - t_s(h_s, \theta_s, v_s) \geq v_s p_s(h_s, \theta_s, v'_s) - t_s(h_s, \theta_s, v'_s),$$

and the individual rationality constraint (IR) is: for any h_s

$$v_s p_s(h_s, \theta_s, v_s) - t_s(h_s, \theta_s, v_s) \geq 0.$$

The feasibility constraint (F) becomes: for every $\bar{h}_S$,

$$\sum_{s=1}^{S} \mathbf{1}_{\{\theta_s = AB\}} \cdot a_s(h_s, v, AB) \equiv Q_{AB}(\bar{h}_S) \leq C_{AB}, \ \sum_{s=1}^{S} \mathbf{1}_{\{\theta_s = BC\}} \cdot a_s(h_s, v, BC) \equiv Q_{BC}(\bar{h}_S) \leq C_{BC},$$

$$\sum_{s=1}^{S} \mathbf{1}_{\{\theta_s = AC\}} \cdot a_s(h_s, v, AB) \leq C_{AC} + \min\{C_{AB} - Q_{AB}(\bar{h}_S), C_{BC} - Q_{BC}(\bar{h}_S)\}.$$

The monopolist problem is to find a dynamic mechanism that maximizes the expected revenue subject to (IC), (IR), and F. The revenue maximization problem can be written recursively as follows: for each $s \in \{1, \dots, S\}$,

$$V_s(C_s) = \max_{p_s} \sum_{\theta} \pi_\theta \int_0^{\bar{v}_\theta} \left[\psi_\theta(v_{\theta,s}) p_s(v_{\theta,s}, \theta) + V_{s+1}(C_{s+1}(p_s(v_{\theta,s}, \theta)))\right] g_\theta(v_{\theta,s}) dv_{\theta,s},$$

where $V_{S+1} \equiv 0$, and $C_{s+1}(p_s(v_{\theta,s}, \theta_s))$ is given by:

(i) For $\theta = AB$, if $p_s(v, AB) = 1$, then $C_{s+1} = (C_{AB,s} - 1, C_{BC,s}, C_{AC,s})$, if $p_s(v, AB) = 0$, then $C_{s+1} = C_s$;
(ii) For $\theta = BC$, if $p_s(v, BC) = 1$, then $C_{s+1} = (C_{AB,s}, C_{BC,s} - 1, C_{AC,s})$, if $p_s(v, BC) = 0$, then $C_{s+1} = C_s$;
(iii) For $\theta = AC$, if $p_s(v, BC) = 1$ and $C_{AC,s} > 0$, then $C_{s+1} = (C_{AB,s}, C_{BC,s}, C_{AC,s} - 1)$, if $p_s(v, BC) = 1$ and $C_{AC,s} = 0$, then $C_{s+1} = (C_{AB,s} - 1, C_{BC,s} - 1, C_{AC,s})$, if $p_s(v, AC) = 0$, then $C_{s+1} = C_s$.

For the subsequent analysis, we also define the monopolist's value function conditional on the arrival of a type θ buyer as:

$$V_s(C_s, \theta) = \max_{p_s} \int_0^{\bar{v}_\theta} \left[\psi_\theta(v_{\theta,s}) p_s(v_{\theta,s}, \theta) + V_{s+1}(C_{s+1}(p_s(v_{\theta,s}, \theta)))\right] g_\theta(v_{\theta,s}) dv_{\theta,s}.$$

The following proposition characterizes the revenue maximizing dynamic mechanism in the relaxed problem.

Proposition 3. *For any s, given a supply vector C_s, the optimal allocation rule allocates objects to meet type θ buyer's demand if her conditional virtual value $\psi_\theta(v_{\theta,s})$ passes a deterministic cutoff $\Delta_{\theta,s}(C_s)$, as long as such an allocation is feasible. These cutoffs are given by:*

(i) *If $C_{AB,s} > 0$, then*

$$\Delta_{AB,s}(C_s) = V_{s+1}(C_{AB,s}, C_{BC,s}, C_{AC,s}) - V_{s+1}(C_{AB,s} - 1, C_{BC,s}, C_{AC,s});$$

otherwise $\Delta_{AB,s}(C_s)$ is any number larger than $\bar{v}_{AB}$.

(ii) *If $C_{BC,s} > 0$, then*

$$\Delta_{BC,s}(C_s) = V_{s+1}(C_{AB,s}, C_{BC,s}, C_{AC,s}) - V_{s+1}(C_{AB,s}, C_{BC,s} - 1, C_{AC,s});$$

otherwise $\Delta_{AB,s}(C_s)$ is any number larger than $\bar{v}_{BC}$.

(iii) *If $C_{AC,s} > 0$, then*

$$\Delta_{AC,s}(C_s) = V_{s+1}(C_{AB,s}, C_{BC,s}, C_{AC,s}) - V_{s+1}(C_{AB,s}, C_{BC,s}, C_{AC,s} - 1);$$

if $C_{AC,s} = 0$ *and* $\min\{C_{AB,s}, C_{BC,s}\} > 0$*, then*

$$\Delta_{AC,s}(C_s) = V_{s+1}(C_{AB,s}, C_{BC,s}, C_{AC,s}) - V_{s+1}(C_{AB,s} - 1, C_{BC,s} - 1, C_{AC,s});$$

otherwise $\Delta_{AC,s}(C_s)$ *is any number larger than* $\bar{v}_{AC}$*.*

Proof. See the Appendix. □

Note that the optimal mechanism for the relaxed problem can be implemented by posted prices: since the cutoffs are in terms of virtual values, the posted prices are the inverse of the virtual value function at the corresponding cutoffs.[16] The next proposition states the properties of these cutoffs.

Proposition 4. *For each* s *and* C_s*, the cutoffs* $\Delta_{\theta,s}(C_s)$ *have the following properties:*

(1) For each $\theta \in \{AC, AB, BC\}$*,* $\triangle_{\theta,s}(C_{AB,s}, C_{BC,s}, C_{AC,s})$ *is weakly decreasing in each argument.*

(2) For $\theta \in \{AB, BC\}$*,* $\triangle_{\theta,s}(C_{AB,s}, C_{BC,s}, C_{AC,s})$ *is weakly decreasing in* $C_{AC,s}$*.*

(3) $\triangle_{AC,s}(C_{AB,s}, C_{BC,s}, C_{AC,s})$ *is weakly decreasing with* $C_{AB,s}$ *and* $C_{BC,s}$*.*

(4) For $\theta, \theta' \in \{AB, BC\}$ *and* $\theta \neq \theta'$*,* $\triangle_{\theta,s}(C_{AB,s}, C_{BC,s}, C_{AC,s})$ *is (weakly) increasing with* $C_{\theta',s}$*.*

Furthermore, for each θ *and* C*,* $\triangle_{\theta,s}(C)$ *is (weakly) decreasing in* s*.*

Proof. See the Appendix. □

The above result implies that over time if no object is assigned, then the cutoffs decrease due to the deadline effect. On the other hand, a sale of a given type of objects has two effects: the first one is a direct effect as now there are fewer units of that object; the second one is an indirect effect on other types of objects. The latter effect is the consequence of the network structure, i.e., the complementarity and substitution among the objects. For instance, after a sale of an AC object, the cutoffs of both AB and BC objects increase, since buyers who want to obtain these objects now face more competition. Interestingly, when a sale of an AB object occurs, the cutoff of BC object weakly decreases, as now BC type buyers face less competition from AC type buyers.

Next we give sufficient conditions under which the posted price mechanism is false-name bid proof. We first consider AB an BC buyers' false-name bidding. The next proposition is a generalization of the result for the static setting.

Proposition 5. *Under Assumptions 1 and 2, in every period* s*,* AB *or* BC *type buyers' false-name proof constraints are satisfied in the solution to the relaxed problem.*

[16] The posted price implementation relies on the assumption that there is only one buyer, or more generally several buyers of the same type, in each period. We show in the next section that if there are multiple types of buyers in each period, the optimal mechanism without the false-name proof constraints cannot be implemented by posted prices.

Proof. There are two cases to check: either $C_{AC,s} > 0$ or $C_{AC,s} = 0$. In the first case, since we know that when a buyer reports AC and there is supply of AC, she will be assigned an AC object. Therefore, in this case neither AB nor BC type buyer will pretend to be an AC buyer.

For the second case, we have for every s,

$$\Delta_{AC,s}(C_{AB}, C_{BC}, 0) \geq \max\{\Delta_{AB,s}(0, C_{AB}, C_{BC}), \Delta_{BC,s}(0, C_{AB}, C_{BC})\},$$

which, together with Assumptions 1 and 2, implies that neither AB nor BC buyers will false-name bid.[17] □

The following condition further guarantees that AC type buyers will never false-name bid in the posted price mechanism.

Assumption 4.

$$\psi_{AB}^{-1}(0) + \psi_{BC}^{-1}(0) \geq \psi_{AC}^{-1}(0),$$

and

$$\inf_{x \in (\psi_{AC}^{-1}(0), \bar{v}_{AC})} \psi'_{AC}(x) \geq \sup_{x \in (\psi_{\theta}^{-1}(0), \bar{v}_{\theta})} \psi'_{\theta}(x),$$

for $\theta \in AB, BC$.

Proposition 6. *Under Assumptions 1, 2 and 4, in every period s, all types of buyers' false-name proof constraints are satisfied in the solution to the relaxed problem.*

Intuitively, Assumption 4 means that the cutoffs in virtual values agree with the cutoffs in values. One way to guarantee this is to assume that AC buyers' virtual value function is always steeper than those of AB and BC buyers. For example, uniform distribution and the exponential distribution satisfy this assumption. On the other hand, Assumption 4 is strong, and it does not hold for many other distributions. Nevertheless, a sub-optimal solution, which is intuitive and false-name proof even without Assumption 4, is to first determine the prices of AB and BC and then set the price of AC to be bounded below by the sum of the prices of AB and BC. This sub-optimal solution is referred to as "bid pricing" in the revenue management literature.

Remark 4. Suppose each buyer's value is drawn from the same distribution, then Assumption 4 can be weakened to the requirement that the virtual value function is convex. Another sufficient condition is that the inverse hazard rate function is decreasing and concave. See the Appendix for the formal proofs of these claims.

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[17]Observe that without Assumption 2, the following is possible: $\Delta_{AC,s}(0, C_{AB}, C_{BC}) \geq \Delta_{AB,s}(0, C_{AB}, C_{BC})$, but $\psi_{AC}^{-1}(\Delta_{AC,s}(0, C_{AB}, C_{BC})) < \psi_{AB}^{-1}(\Delta_{AB,s}(0, C_{AB}, C_{BC}))$, then the AB buyer may have incentive to false-name bid as an AC buyer.

4. The Dynamic Problem with False-name Bidding

In this section we focus on buyers' false name bidding incentives and examine cases in which the assumptions in the previous section do not hold. In order to have non-trivial false-name bidding possibilities, we need to modify the setting such that there are multiple buyers in any period. For simplicity, we assume that in each period the seller either meets with an AC buyer with probability ρ or meets with an AB buyer and a BC buyer with probability $1-\rho$.

False-name bidding by AB and BC type buyers. We first consider false-name biddings by AB buyers. Note that the analysis of the last period will be the same as that in the static setting of case 1 in Section 2.2. Suppose that the monopolist has one unit of AB and one unit of BC.[18] For simplicity, assume $S=2$. From the analysis of the static model, in the last period we have $t_{AB,2}=t_{BC,2}=t_{AC,2}\equiv t_2$. Therefore the cutoff for AC at $s=1$ is given by

$$\triangle_{AC,1}=t_2\left[\rho_{AC}(1-G_{AC}(t_2))+\rho_{AB}(1-G_{AB}(t_2))+\rho_{BC}(1-G_{BC}(t_2))\right].$$

That is, if the AC buyer's virtual value is above the cutoff then both goods are assigned to her. Compared to the case in which the false-name proof constraint in the second period is ignored, the cutoff $\triangle_{AC,1}$ is smaller.[19] Similarly, the cutoffs for AB and BC at $s=1$ are defined as follows

$$\begin{aligned}\triangle_{AB,1}&=t_2\left[\rho_{AC}(1-G_{AC}(t_2))+\rho_{AB}(1-G_{AB}(t_2))+\rho_{BC}(1-G_{BC}(t_2))\right]\\&\quad-t^*_{BC,2}[\rho_{BC}(1-G_{BC}(t^*_{BC,2}))],\\\triangle_{BC,1}&=t_2\left[\rho_{AC}(1-G_{AC}(t_2))+\rho_{AB}(1-G_{AB}(t_2))+\rho_{BC}(1-G_{BC}(t_2))\right]\\&\quad-t^*_{AB,2}[\rho_{AB}(1-G_{AB}(t^*_{AB,2}))],\end{aligned}$$

where $t^*_{AB,2}$ and $t^*_{BC,2}$ denote the unconstrained (i.e., without false-name proof constraints) optimal transfers for AB and BC objects in the second period, respectively.

The key point is that, after assigning the AB object in first period, there is no false-name proof constraint in second period: an AB buyer can no longer imitate an AC buyer. Compared to the unconstrained case, $\triangle_{AB,1}$ is smaller, since $t^*_{AB,2}>t_{AB,2}$. As an implication, the false-name proof mechanism in the second period mitigates AB and BC buyers' false-name bidding incentives in the first period. Since we have $\triangle_{AC,1}>\max\{\triangle_{AB,1},\triangle_{BC,1}\}$, compared to the static case in which all the cutoffs in term of virtual values are zero, AB and BC buyers' false-name bidding incentives are also weaker in the dynamic case.

[18] Notice that if there is also an AC unit available, then AB and BC buyers will never try to pretend as an AC buyer in period 1.

[19] Notice that each term in $\Delta_{AC,1}$ is larger when false-name proof constraint is ignored.

Finally, note that false-name proof constraints can still bind in the first period, since it is possible to have $\triangle_{AB,1} < \triangle_{AC,1}$ but $\psi_{AB}^{-1}(\triangle_{AB,1}) \geq \psi_{AC}^{-1}(\triangle_{AC,1})$. When it binds, by similar arguments as in section 3.3, the monopolist solves the following problem

$$\max_{c_{AB,1},c_{BC,1},c_{AC,1}} \rho_{AC} \int_{c_{AC,1}}^{\bar{v}_{AC}} (\psi_{AC}(v_{AC}) - \triangle_{AC,1})\, g_{AC}(v_{AC}) dv_{AC}$$
$$+ \rho_{AB} \int_{c_{AB,1}}^{\bar{v}_{AB}} (\psi_{AB}(v_{AB}) - \triangle_{AB,1})\, g_{AB}(v_{AB}) dv_{AB}$$
$$+ \rho_{BC} \int_{c_{BC,1}}^{\bar{v}_{BC}} (\psi_{BC}(v_{BC}) - \triangle_{BC,1})\, g_{BC}(v_{BC}) dv_{BC}$$

subject to $c_{AC,1} \geq c_{AB,1}$ and $c_{AC,1} \geq c_{BC,1}$.

False-name bidding by AC type buyers. Next we extend Case 2 in Section 2.2 to the dynamic setting. Suppose again that the monopolist has one unit of AB and one unit of BC, and $S = 2$. We first analyze the relaxed problem in which false-name proof constraints are ignored. Define the expected continuation payoffs α, β and γ as follows:

$$\alpha = \rho \mathbf{E}[\max\{0, \psi_{AC}(v_{AC})\}] + (1-\rho)\{\mathbf{E}[\max\{0, \psi_{AB}(v_{AB})\}] + \mathbf{E}[\max\{0, \psi_{BC}(v_{BC})\}]\}$$
$$\beta = (1-\rho)\mathbf{E}[\max\{0, \psi_{AB}(v_{AB})\}]$$
$$\gamma = (1-\rho)\mathbf{E}[\max\{0, \psi_{BC}(v_{BC})\}]$$

Given α, β and γ, in period 1 the monopolist's problem conditional on the arrival of AB and BC buyers is:

$$\max_{p_{AB,1},p_{BC,1}} \int_0^{\bar{v}_{AB}} \int_0^{\bar{v}_{BC}} \Big[p_{AB,1}(v_{AB}, v_{BC})\psi_{AB}(v_{AB}) + p_{BC,1}(v_{AB}, v_{BC})\psi_{BC}(v_{BC})$$
$$p_{AB,1}(v_{AB}, v_{BC})(1 - p_{BC,1}(v_{AB}, v_{BC}))\gamma + p_{BC,1}(v_{AB}, v_{BC})(1 - p_{AB,1}(v_{AB}, v_{BC}))\beta$$
$$+(1 - p_{AB,1}(v_{AB}, v_{BC}))(1 - p_{BC,1}(v_{AB}, v_{BC}))\alpha \Big] g_{AB}(v_{AB}) g_{BC}(v_{BC}) dv_{AB} dv_{BC}.$$

The problem can be simplified to:

$$\max_{p_{AB,1},p_{BC,1}} \int_0^{\bar{v}_{AB}} \int_0^{\bar{v}_{BC}} \Big[p_{AB,1}(v_{AB}, v_{BC})(\psi_{AB}(v_{AB}) + \gamma - \alpha) + p_{BC,1}(v_{AB}, v_{BC})(\psi_{BC}(v_{BC}) + \beta - \alpha)$$
$$p_{AB,1}(v_{AB}, v_{BC}) p_{BC,1}(v_{AB}, v_{BC})(\alpha - \beta - \gamma) \Big] g_{AB}(v_{AB}) g_{BC}(v_{BC}) dv_{AB} dv_{BC},$$

which can be solved by point-wise maximization.

For simplicity, consider the symmetric case where $g_{AB} = g_{BC}$. Thus we have $\beta = \gamma$. Also note that $\alpha > \beta + \gamma$. The optimal allocations $p_{AB,1}$ and $p_{BC,1}$ are given by:

$$p_{AB,1}(v_{AB}, v_{BC}) = p_{BC,1}(v_{AB}, v_{BC}) = 1, \quad \text{if } \psi_{AB}(v_{AB}) + \psi_{BC}(v_{BC}) \geq \alpha \text{ and } \psi_{AB}(v_{AB}) \geq \beta,$$

$$p_{AB,1}(v_{AB}, v_{BC}) = 1,\ p_{BC,1}(v_{AB}, v_{BC}) = 0, \quad \text{if } \psi_{AB}(v_{AB}) \geq \alpha - \gamma \text{ and } \psi_{BC}(v_{BC}) < \gamma,$$
$$p_{AB,1}(v_{AB}, v_{BC}) = 0,\ p_{BC,1}(v_{AB}, v_{BC}) = 1, \quad \text{if } \psi_{BC}(v_{BC}) \geq \alpha - \beta \text{ and } \psi_{AB}(v_{AB}) < \beta,$$
$$p_{AB,1}(v_{AB}, v_{BC}) = p_{BC,1}(v_{AB}, v_{BC}) = 0, \quad \text{otherwise.}$$

See Figure 4.1 for an illustration of the optimal allocations.

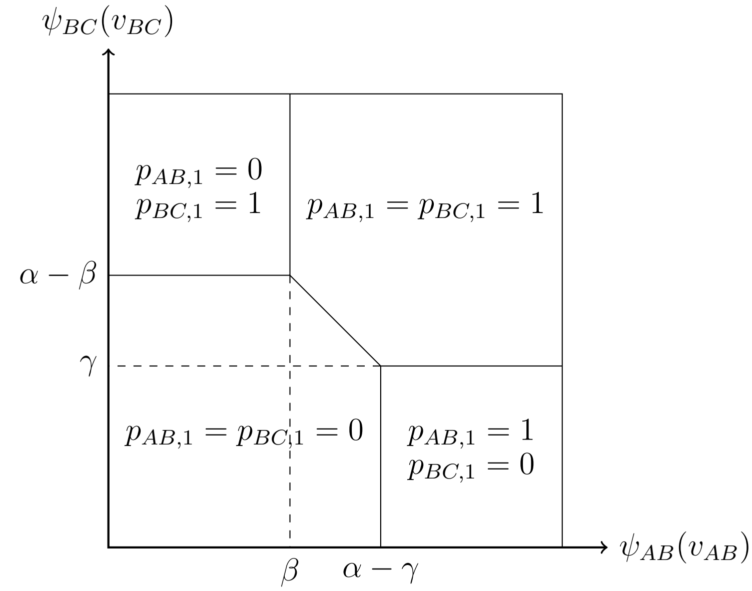

FIGURE 4.1. Optimal allocation in period 1 without false-name bidding.

In the optimal mechanism without false-name bidding, $p_{AB,1}(v_{AB}, v_{BC}) = 1$ whenever $\psi_{AB}(v_{AB}) > \alpha - \gamma$; $p_{BC,1}(v_{AB}, v_{BC}) = 1$ whenever $\psi_{BC}(v_{BC}) > \alpha - \beta$. It follows that if both ψ_{AB} and ψ_{BC} are increasing functions then the interim allocation rules $P_{AB,1}(v_{AB})$ and $P_{BC,1}(v_{BC})$ are both non-decreasing, where

$$P_{AB,1}(v_{AB}) = \int_0^{\bar{v}} p_{AB,1}(v_{AB}, v_{BC}) g_{BC}(v_{BC}) dv_{BC}$$

and

$$P_{BC,1}(v_{BC}) = \int_0^{\bar{v}} p_{BC,1}(v_{AB}, v_{BC}) g_{AB}(v_{AB}) dv_{AB},$$

and hence the mechanism is Bayesian incentive compatible. Note that even though the (ex post) allocation rules are deterministic, they are not cutoff rules; as a result, from either AB or BC buyer's view, the interim allocation rules involve randomization.

Note that the above mechanism cannot be implemented by posted prices: the price that an AB buyer needs to pay for the AB good varies with the reports of the BC buyer, and vice versa. For instance, the corresponding (ex post incentive compatible and individually rational) transfer rule for the AB buyer is described as follows.

- If $v_{BC} < \psi_{BC}^{-1}(\gamma)$, then

$$t_{AB,1}(v_{AB}, v_{BC}) = \begin{cases} \psi_{AB}^{-1}(\alpha - \gamma), & \text{if } v_{AB} > \psi_{AB}^{-1}(\alpha - \gamma) \\ 0, & \text{otherwise;} \end{cases}$$

- If $\psi_{BC}^{-1}(\gamma) < v_{BC} < \psi_{BC}^{-1}(\alpha - \beta)$, then

$$t_{AB,1}(v_{AB}, v_{BC}) = \begin{cases} \psi_{AB}^{-1}(\alpha - \psi_{BC}(v_{BC})), & \text{if } \psi_{AB}(v_{AB}) + \psi_{BC}(v_{BC}) > \alpha \\ 0, & \text{otherwise;} \end{cases}$$

- If $v_{BC} > \psi_{BC}^{-1}(\alpha - \beta)$, then

$$t_{AB,1}(v_{AB}, v_{BC}) = \begin{cases} \psi_{AB}^{-1}(\beta), & \text{if } v_{AB} > \psi_{AB}^{-1}(\beta) \\ 0, & \text{otherwise.} \end{cases}$$

Now consider the possibility of false-name bidding by the AC buyer in period 1. If in the second period the false-name proof constraint does not bind ($\psi_{AC}^{-1}(0) < \psi_{AB}^{-1}(0) + \psi_{BC}^{-1}(0)$), then α, β and γ remain the corresponding expected discounted surplus for the monopolist from the second period when both AB and BC are available, only AB is available, and only BC is available, respectively. Since the allocation rules $p_{AB,1}$ and $p_{BC,1}$ are deterministic, the AC buyer will report v_{AB} and v_{BC} such that $p_{AB,1}(v_{AB}, v_{BC}) = p_{BC,1}(v_{AB}, v_{BC}) = 1$. It is possible to have $\psi_{AC}^{-1}(\alpha) > \psi_{AB}^{-1}(\beta) + \psi_{BC}^{-1}(\gamma)$. This implies that even if in the static model AC's false-name proof constraint does not bind, because of the possibility of dynamic allocations, now AC's false-name proof constraint binds in the first period. Thus, the optimal deterministic mechanism distorts the first period's cut-offs until the false-name proof constraint holds with equality.

Finally, recall that in the static setting, randomized allocations can outperform deterministic allocations. This insight extends to the dynamic case; nevertheless AC's incentive to false-name bid in the first period always gets stronger regardless of the allocation rules in the second period.

5. Extensions

5.1. **General Network.** Our baseline model can be extended to a more general network structure, albeit the assumptions should be modified. Here we give an outline of the graph representation of a general heterogeneous objects model. A directed graph G with N nodes can be represented by an $N \times N$ matrix with entries either one or zero, indicating whether there is an edge between any two nodes. A path q is sequence of distinct edges (without cycles) that has an origin o and a destination d. Buyer's type is modeled as the (v, o, d) triplet. In the context of the airline pricing problem, the interpretation is that buyer i wants to obtain any subsets of objects with which she can travel from her departure city o to her destination city d. We conjecture that the results for general networks will be qualitatively similar to the simple network structure studied in this paper. In particular, the optimal mechanism in general involves randomized allocations.

5.2. **The Discounted Case.** The model can be extended to cover the following possibility. Suppose that each buyer type is described by a favorite object, an acceptable set of objects, and a value. Following our airline pricing example, consider an AC type buyer, if she gets a direct flight (her favorite object), she will get utility v (her value); if she gets any indirect flight (one of the acceptable objects), then she will get δv (her discounted value), where $\delta \in (0,1)$. If each buyer's favorite object and the discount factor $\delta \in (0,1)$ are public knowledge (we explicitly assume that the favorite object is the shortest path connecting the origin and the destination), then it is straightforward to extend our results to this case. On the other hand, if each buyer's discount factor is her private knowledge, then there are additional incentive constraints, i.e., each buyer should report her discount factor truthfully. We leave this possibility for future research.

5.3. **Patient Buyers.** The problem becomes more complicated if buyers are long-lived. One simple case is that the seller and buyers do not discount the future, yet the time horizon is still finite. Then the dynamic problem is equivalent to the static problem. The seller will not allocate any good prior to the deadline S, and at the deadline S he will implement the static mechanism. In the case where the seller discounts the future and buyers report their arrival times, if we impose the assumption that those buyers who arrive earlier have higher values according to our order over types, then by Proposition 3 of Vohra and Pai (2013a) we can accommodate the strategic reporting of the arrival times (note that the deadline is public in our model). Alternatives, is having an observed arrival time then buyers report their deadline as in Mienderoff (2015).

5.4. **Multiple Buyers in A Dynamic Model.** Suppose we maintain the assumption that there is only one type of buyers in every period, but the number of buyers can be arbitrary. Specifically, for each θ, the number of corresponding buyers L_θ is a discrete random variable. At the beginning of each period s, the corresponding demand is denoted by $L_{\theta,s}$. If one ignores the false-name proof constraint, then this case is a straightforward extension of the setting in Section 4. Unfortunately, this case becomes much more complicated when false-name bidding is possible. Nevertheless, in this case false-name bidding incentives are mitigated due to the competition between agents. That is, the false-name bidding buyer has to take in to account the expected value of the other agents. Solving for the optimal mechanism in the multiple buyers case is a difficult task. This question is left for future research.

Appendix A. Proofs

Proof of Proposition 2:

Proof. The first part of Proposition 2 is standard, hence we omit the proof. Also notice that without loss of generality one can focus on exact allocation rules. Exact allocation rules are the ones never assigns an object which is not in the acceptance set of the agent. Consider an AB type buyer with value v. Her false-name proof constraint is:

$$vP(v, AB) - T(v_{AB}, AB) \geq vP(v, AC) - T(v, AC).$$

From the AC type buyer's incentive compatibility constraint, we have

$$vP(v, AC) - T(v, AC) \geq vP(v', AC) - T(v', AC).$$

Adding these two constraints yields

$$vP(v, AB) - T(v_{AB}, AB) \geq vP(v', AC) - T(v', AC).$$

Finally, since the mechanism does not pay subsidy, the result follows. □

Lemma 1. The optimal dynamic allocation rule without false-name proof constraints is a cutoff rule.

Proof. Given a supply vector C_s in period s, suppose by contradiction that a buyer with private information (v, θ) fulfills her demand but a same type θ buyer with a higher value $v' > v$ does not. Then by optimality, we have

$$\psi_\theta(v) + V_{s+1}(p_s(C_s, v, \theta)) \geq V_{s+1}(C_s)$$

and

$$\psi_\theta(v') + V_{s+1}(p_s(C_s, v', \theta)) < V_{s+1}(C_s).$$

Since $p_s(C_s, v, \theta) = p_s(C_s, v', \theta)$, adding up the above two inequalities yields $\psi_\theta(v') - \psi_\theta(v) < 0$, which is a contradiction. □

Next we state and prove the following lemma, which generalizes Lemma 2-2 A1 in Talluri and van Ryzin(2004). For any increasing function $\gamma : \mathbb{N} \to \mathbb{R}$, we say $\gamma(n)$ is *concave* if for any $n > 1$,

$$\gamma(n + 1) - \gamma(n) \leq \gamma(n) - \gamma(n - 1).$$

Lemma 2. Suppose $\xi : \mathbb{N}^3 \to \mathbb{R}$ is concave in each argument (keeping other arguments constant). Let $\eta : \mathbb{N}^3 \to \mathbb{R}$ be defined by

$$\eta\left(x_1, x_2, x_3\right) = \max_{a=0,1,\ldots,m} \left\{ \sum_{i=0}^{a} p_i + \xi\left(x_1 - a, x_2, x_3\right) \right\}$$

for some given non-negative sequence $(p_i)_{i=0}^{x_1}$ with $p_0 = 0$, $p_1 \geq p_2 \geq \cdots \geq p_{x_1} \geq 0$ and $m \leq x_1$. Then η is also concave in each argument (keeping other arguments constant).

Proof. This follows from the following inequality: for any $a \geq 1$,

$$\begin{aligned}
&\left(\xi(x_1 - a + 1, x_2, x_3) + \sum_{i=1}^{a-1} p_i\right) - \left(\xi(x_1 - a, x_2, x_3) + \sum_{i=1}^{a} p_i\right) \\
&= \xi(x_1 - a + 1, x_2, x_3) - \xi(x_1 - a, x_2, x_3) - p_a \\
&\leq \xi(x_1 - a, x_2, x_3) - \xi(x_1 - a - 1, x_2, x_3) - p_{a+1} \\
&= \left(\xi(x_1 - a, x_2, x_3) + \sum_{i=1}^{a} p_i\right) - \left(\xi(x_1 - a - 1, x_2, x_3) + \sum_{i=1}^{a+1} p_i\right).
\end{aligned}$$

□

Lemma 3. In the optimal mechanism, for any $s \in \{1, \ldots, S\}$, when $p_{AC,s} = 1$, the corresponding AC buyer is assigned with an AC object if $C_{AC,s} > 0$, otherwise she is assigned an AB object and a BC object.

Proof. We want to show that when $C_{AC,s} > 0$, $V_s(C_{AB,s}, C_{BC,s}, C_{AC,s} - 1) \geq V_s(C_{AB,s} - 1, C_{BC,s} - 1, C_{AC,s})$ for all s. The proof is by backward induction. At $s = S$ the result holds by definition. For any $s < S$ and v_{AC}, suppose $C_{AC,s} > 0$. Define

$$H(C_s, v_{AC}) = \psi_{AC}(v_{AC}) + V_{s+1}(C_{AB,s}, C_{BC,s}, C_{AC,s} - 1)$$

and

$$H(C_s, v_{AC}) = \psi_{AC}(v_{AC}) + V_{s+1}(C_{AB,s} - 1, C_{BC,s} - 1, C_{AC,s})$$

By the induction hypothesis, we have $H \geq D$. since this holds for all v_{AC} such that $p_s(v_{AC}, AC) = 1$ and if $p_s(v_{AC}, AC) = 0$ then $C_{s+1} = C_s$, the result follows. □

Next we prove Propositions 3 and 4 in Section 3.

Proof of Propositions 3:

Proof. The result follows from Lemma 1 and Lemma 3. □

Proof of Propositions 4:

Proof. (1) We want to show that for any $s \in \{1, \ldots, S\}$ and $\theta \in \{AB, BC, AC\}$, when $C_{\theta,s}$ increases, $\Delta_{\theta,s}(C_s)$ weakly decreases. Here we show that this holds for $C_{AC,s}$. The proofs for $C_{AB,s}$ and $C_{BC,s}$ are the same. We proceed by backward induction. First, recall that $V_{S+1} = 0$, thus the claim holds for $s = S$. Next assume that V_{s+1} satisfies the property and consider period s.

$$\begin{aligned} & V_s(C_s, AC) \\ =& \max_{p_s(C_s, v_{AC}, AC)} \int_0^{\bar{v}_{AC}} \left[\psi_{AC}(v_{AC}) p_s(C_s, v_{AC}, AC) + V_{s+1}(C_{s+1}(p_s(C_s, v_{AC}, AC)))\right] g_{AC}(v_{AC}) dv_{AC} \\ =& \int_0^{\bar{v}_{AC}} \max_{p_s \in \{0,1\}} \left[\psi_{AC}(v_{AC}) p_s + V_{s+1}(C_{AB,s}, C_{BC,s}, C_{AC,s} - p_s)\right] g_{AC}(v_{AC}) dv_{AC} \end{aligned}$$

Note that the term inside the integral on the right-hand-side of the value function has the same form as in Lemma 4, thus it is concave in C_{AC}. It follows that $V_s(C_s, AC)$ is also concave in $C_{AC,s}$. The result then holds by the definition of $\Delta_{\theta,AC}(C_s)$.

(2) We use induction to show that $\triangle_{AB,s}(C_{AB,s}, C_{BC,s}, C_{AC,s} - 1) \geq \triangle_{AB,s}(C_{AB,s}, C_{BC,s}, C_{AC,s})$. At time S the inequality holds trivially. Suppose the inequality holds for $s+1$, we now show that it holds for s. Similar to the previous argument, for each v_{AB}, define,

$$H_s(C_s, v_{AB}) \equiv \max_{p_s \in \{0,1\}} \left[\psi_{AB}(v_{AB}) p_s + V_{s+1}(C_{AB,s} - p_s, C_{BC,s}, C_{AC,s})\right].$$

Then for each s,

$$H_s(C_s, v_{AB}) = V(C_s) + \max\{0, \psi_{AB}(v_{AB}) - \Delta_{AB,s}(C_s)\}.$$

Therefore, we have

$$H_{s+1}(C_{s+1}, v_{AB}) = V(C_{s+1}) + \max\{0, \psi_{AB}(v_{AB}) - \Delta_{AB,s+1}(C_{s+1})\},$$

and

$$\begin{aligned} & H_{s+1}(C_{AB,s+1}, C_{BC,s+1}, C_{AC,s+1} - 1, v_{AB}) \\ =& V(C_{AB,s+1}, C_{BC,s+1}, C_{AC,s+1} - 1) + \max\{0, \psi_{AB}(v_{AB}) - \Delta_{AB,s+1}(C_{AB,s+1}, C_{BC,s+1}, C_{AC,s+1} - 1)\}. \end{aligned}$$

Since

$$\Delta_{AB,s}(C_{AB,s+1}, C_{BC,s+1}, C_{AC,s+1} - 1) \geq \Delta_{AB,s}(C_{s+1}),$$

and

$$V(C_s) \geq V(C_{AB,s+1}, C_{BC,s+1}, C_{AC,s+1} - 1),$$

it follows that for each v_{AB},

$$H_{s+1}(C_{s+1}, v_{AB}) \geq H_{s+1}(C_{AB,s+1}, C_{BC,s+1}, C_{AC,s+1} - 1, v_{AB}).$$

The result then follows from taking expectation with respect to v_{AB}.

(3) We need to show $\Delta_{AC,s}(C_{AB} - 1, C_{BC}, C_{AC}) \geq \Delta_{AC,s}(C_{AB}, C_{BC}, C_{AC})$. The proof uses the same argument as in case 2 by induction.

(4) We need to show $\Delta_{AB,s}(C_{AB}, C_{BC} - 1, C_{AC}) \geq \Delta_{AB,s}(C_{AB}, C_{BC}, C_{AC})$. The proof again follows the same argument as in case 2 by induction.

(5) We prove monotonicity of $\Delta_{AC,s}$ with respect to s. The proofs for the other two cutoffs are the same. First recall that

$$V_s(C) = \sum_\theta \pi_\theta \left\{ \int_0^{\bar{v}_\theta} \max_{p_s \in \{0,1\}} \left[p_s \psi_\theta(v_\theta) + V_{s+1}(C_\theta - p_s, C_{-\theta}) \right] g_\theta(v_\theta) dv_\theta \right\},$$

where $C_{-\theta} = \{C_{AB}, C_{BC}, C_{AC}\} \setminus \{C_\theta\}$. Then

$$\begin{aligned}
\Delta_{AC,s}(C) &= V_{s+1}(C) - V_{s+1}(C_{AB}, C_{BC}, C_{AC} - 1) \\
&= \Delta_{AC,s+1}(C) + \sum_\theta \pi_\theta \left\{ \int_0^{\bar{v}_\theta} \left[\max\{0, \psi_\theta(v_\theta) - \Delta_{\theta,s+1}(C)\} \right.\right. \\
&\qquad\qquad \left.\left. - \max\{0, \psi_\theta(v_\theta) - \Delta_{\theta,s+1}(C_{-AC}, C_{AC} - 1)\} \right] g_\theta(v_\theta) dv_\theta \right\} \\
&\geq \Delta_{AC,s+1}(C)
\end{aligned}$$

The last inequality follows from the fact that for each θ, $\Delta_{\theta,s+1}(C_{AC} - 1, C_{AB}, C_{BC}) \geq \Delta_{\theta,s+1}(C)$.

□

Finally, the following lemma proves the claim in Remark 4 in Section 3.

Lemma 4. If $g_{AC}(v) = g_{AB}(v) = g_{BC}(v) = g(v)$ almost surely, then convexity of the virtual value function $\psi(v) = v - \frac{1-G(v)}{g(v)}$ is a sufficient condition for the optimal mechanism for the relaxed problem to be false-name proof.

Proof. We want to show that for any x, y, z with the properties that $\max(\psi(x), \psi(y)) \leq \psi(z)$ and $\psi(x) + \psi(y) \geq \psi(z)$, we have $x + y > z$. A sufficient condition for this to hold is that the function ψ is supper-additive. Too see this, consider the contrapositive of the statement, i.e., whenever $\max(\psi(x), \psi(y)) \leq \psi(z)$, $x + y \leq z$ implies $\psi(x) + \psi(y) < \psi(z)$. Since ψ is an increasing function, the above statement follows. Note that since $\psi(0) \leq 0$, if ψ is a convex function, then it is also super-additive. Finally, convexity of ψ means

$$\psi''(x) = \frac{d}{dx} \left[\frac{g'(x)(1 - G(x))}{g^2(x)} \right] \geq 0,$$

which reduces to the following condition on g:

$$g''(x) > \frac{2(g'(x))^2(1 - G(x) + g(x)/2)}{g(x)(1 - G(x))} = \frac{g'(x)^2}{1 - G(x)} + \frac{2(g'(x))^2}{g(x)}.$$

□

Appendix B. Derivation of the generalized virtual values in Section 2.3

Using integration by parts and rearranging, we have

$$\int_0^{\bar{v}} \lambda(v) \int_0^v \left[p_{AB}(s) - p_{AC}(s)\right] ds dv$$

$$= \int_0^{\bar{v}} \Lambda(v) p_{AB}(v) dv - \Lambda(v) \int_0^v p_{AB}(s) ds \Big|_0^{\bar{v}} - \int_0^{\bar{v}} \Lambda(v) p_{AC}(v) dv + \Lambda(v) \int_0^v p_{AC}(s) ds \Big|_0^{\bar{v}}$$

$$= \int_0^{\bar{v}} \Lambda(v) p_{AB}(v) dv - \int_0^{\bar{v}} \Lambda(v) p_{AC}(v) dv$$

where

$$\int_0^{\bar{v}} p_{AB}(v) \left[(1-\rho) f_{AB}(v) \psi_{AB}(v) + \Lambda(v)\right] dv = (1-\rho) \int_0^{\bar{v}} p_{AB}(v) \left[\psi_{AB}(v) + \phi_{AB}(v)\right] f_{AB}(v) dv,$$

where $\phi_{AB}(v) = \frac{\Lambda(v)}{(1-\rho) f_{AB}(v)}$. Likewise

$$\int_0^{\bar{v}} p_{AC}(v) \left[\rho f_{AC}(v) \psi_{AC}(v) - \Lambda(v)\right] dv = \rho \int_0^{\bar{v}} p_{AC}(v) \left[\psi_{AC}(v) - \phi_{AC}(v)\right] f_{AC}(v) dv,$$

where $\phi_{AC}(v) = \frac{\Lambda(v)}{\rho f_{AC}(v)}$.

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